DOCUMENT RESUME

ED 370 872 SO 023 987

AUTHOR	Gross, Fred E.; And Others
TITLE	The Power of Numbers. A Teacher's Guide to Mathematics in a Social Studies Context. An Interdisciplinary Curriculum.
INSTITUTION	Educators for Social Responsibility, Cambridge, MA.
REPORT NO	ISBN-0-942349-05-9
PUB DATE	93
NOTE	277p.
AVAILABLE FROM	Educators for Social Responsibility, 23 Garden St., Cambridge, MA 02138.
PUB TYPE	Guides - Classroom Use - Teaching Guides (For Teacher) (052) -- Books (010)
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	Census Figures; Citizenship Education; Decision Making; *Demography; *Interdisciplinary Approach; Learning Activities; *Mathematics Instruction; Problem Solving; Secondary Education; *Social Studies; *Statistics; Thinking Skills

ABSTRACT

This document is the teacher's guide for a curriculum designed to teach mathematics in a social studies context. It provides mathematical experiences in real world contexts that help students interpret, experiment, communicate, and look for multiple solutions to complex problems. The curriculum uses mathematics in context to help students develop higher order thinking and communication skills. This approach encourages students to interact with each other and the material. Students begin to see that skills and concepts are part of a connected body of knowledge. The context gives students a reason to learn and remember mathematical skills and shows them how these skills and concepts are applied in actual practice. It shows them that they can use mathematics for learning about the world. This book provides a variety of thematic contexts for mathematical skills and mathematical understanding, including polling, studying trends in census data, and designing a public rail transportation system for Los Angeles. Through these activities students participate in discussions, interpret and analyze data, make decisions, and present their ideas. In the context of thematic applications, students see how otherwise apparently unrelated skills are related. They learn about the interdisciplinary nature of real problems. Each of the two parts of the guide is divided into topical chapters. Each chapter starts with an overview and a list of mathematical concepts, skills, vocabulary, and materials involved in the activities in that chapter. Each activity includes a list of handouts that are located at the end of each chapter. Contains 20 references. (DK)

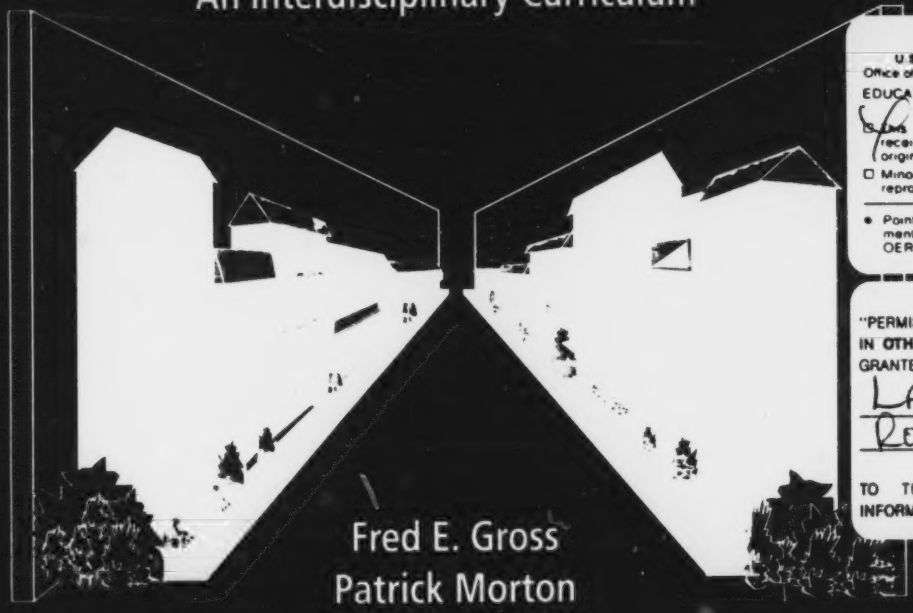
****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED 370 872

THE POWER OF NUMBERS

A Teacher's Guide to Mathematics in a Social Studies Context

An Interdisciplinary Curriculum



U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

- This document has been reproduced as received from the person or organization originating it.
- ☐ Minor changes have been made to improve reproduction quality.
- Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL IN OTHER THAN PAPER COPY HAS BEEN GRANTED BY
LAURA PARKER ROERDEN
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

Fred E. Gross
Patrick Morton
Rachel A. Poliner

BEST COPY AVAILABLE

THE POWER OF NUMBERS

A Teacher's Guide to Mathematics in a Social Studies Context

An Interdisciplinary Curriculum

Fred E. Gross
Patrick Morton
Rachel A. Poliner

Educators for Social Responsibility
Cambridge, Massachusetts

Educators for Social Responsibility
23 Garden Street
Cambridge, MA 02138

© 1993 Boston Area Educators for Social Responsibility
and Fred E. Gross, Patrick Morton, and Rachel A. Poliner.
All rights reserved.
Printed in the United States of America
99 98 97 96 95 94 93 7 6 5 4 3 2 1

ISBN 0-942349-05-9

Cover design by Karen Porti!

Contents

Student Handouts *vii*

Acknowledgments *ix*

Introduction *xi*
- Why Mathematics in Context? *xii*
- Thinking and Communication Skills *xiii*
- How to Use This Book *xiv*
- Don't Miss the Appendices *xiv*

Math Skills by Section *xvii*

Part 1 Counting on Opinions

Introduction *3*

A Poll about Young People *7*
- Activity 1: A Study of Young People, "Monitoring the Future" *8*
- Activity 2: A Short Poll from "Monitoring the Future" *9*

What Does This Have to Do with Math? or Who Needs Math Skills in Social Studies? *15*
- Activity 1: What is Math? *16*
- Activity 2: What Does Math Have to Do with Social Studies? *17*

Choosing a Topic *21*
- Activity: Deciding on a Topic *22*

The Art of Questioning *25*
- Activity 1: Class Discussion on Biased and Unbiased Questions *26*
- Activity 2: Bias in Context *28*
- Activity 3: The Mechanics of Questions *28*
- Activity 4: Length of the School Year *32*
- Activity 5: Writing Questions *32*

Nuts and Bolts of Statistics *37*
- Activity 1: Populations and Samples *38*
- Activity 2: Sampling Experiment *41*
- Going Further with Sampling: Fishcounting *43*

Activity 3: Randomness 46
Going Further with Randomness 50
Activity 4: Random Samples 51
Activity 5: Pick a Sample 53

Designing and Conducting Your Own Poll 65
Class Discussion: Designing a Poll 65
Activity: Constructing a Poll 66

From Highs to Lows: An Average Tour 69
Activity 1: Introduction to Averages 70
Activity 2: Averages and Frequency Distributions 72
Activity 3: Mean Average 74
Activity 4: Diversity in School Population 74
Activity 5: The Neighborhood Restaurant 75
Activity 6: The Average Arbitrator 76

Experimenting with Graphs 81
Introducing Graph Types 81
Activity 1: Line Graphs 83
Activity 2: Circle Graphs 83
Activity 3: Bar Graphs 83
Activity 4: Picture Graphs 84
Activity 5: Graphing Population Density 85
Activity 6: Comparing Graphs 86
Activity 7: Scaling Line and Bar Graphs 86
Activity 8: Choosing and Applying Averages and Graphs 87

Interpreting Polls 99
Activity 1: Introducing Interpretation 101
Activity 2: Assessing the Process: "Portrait of a Generation" 102
Activity 3: Opposing Points of View 103
Activity 4: Margin of Error 104
Activity 5: Final Project 106

Part 2 Counting on People

Introduction 119

Classroom Census 121
Activity: Taking a Class Census 122

The Census: History and Controversy 123
Activity 1: Class Discussion 124
Going Further 124

Activity 2: Estimating Enumerators *124*
Going Further *127*

The Census and Reapportionment *135*
Activity 1: Background *136*
Activity 2: Average District Size *137*
Activity 3: Reapportioning in the Country of Frachat *137*
Activity 4: Reapportioning by the Webster Method *138*
Activity 5: Reapportioning by the Hamilton-Vinton Method *140*
Activity 6: Comparing the Two Methods *141*
Going Further with the Webster Method *144*
Activity 7: The Effects of Undercounting on Reapportionment *147*

The Census and Redistricting for the House of Representatives *163*
Activity 1: Redistricting in Billows *164*
Activity 2: Districts in Your State and Population Density *164*

Rates of Change *175*
Activity 1: Opening Class Discussion: Comparing Changes in Population *176*
Activity 2: Class Discussion: Finding Average Rates of Change *178*
Activity 3: Algebra Exercise: Recognizing Patterns *179*
Activity 4: Formula Making *180*
Going Further *181*
Activity 5: Graphing *182*
Activity 6: Rates and Trends over Time *183*
Going Further *184*

Doubling Time *191*
Activity 1: Understanding Doubling Time *192*
Activity 2: Reaction Time to Exponential Change *192*
Activity 3: Doubling Time of the U.S. Population *193*
Going Further *194*

Cross-Referencing Census Data *201*
Activity 1: Cross-Referencing *202*
Activity 2: Research Interviews *205*
Activity 3: Extrapolating New Data *205*

Creating a Public Rail Transportation System for Los Angeles County *209*
Introduction *209*
Activity 1: Mapping It Out *210*
Activity 2: Developing a Preliminary Plan *212*

The L.A. Transportation Project, Option 1: Cost Factors *221*
Activity 1: Estimating the Cost of Construction *222*
Activity 2: Staying within a Budget *222*

The L.A. Transportation Project, Option 2: Traveling on Time 229
Activity 1: Distance, Rate, and Time (Formula Making) 230
Activity 2: Pacing against Time 231
Activity 3: Graphing Time and Rate vs. Distance 232
Activity 4: Traveling on Time 232
Going Further 233

Appendices

Appendix A: NCTM Goals 243

Appendix B: Cooperative Learning Techniques and *The Power of Numbers* 247
Getting into Groups 247
Helping Small Groups Work Well 248
For Discussions or Presentations with the Whole Class 248
Building Interdependence and Accountability 249
Selected Bibliography for Further Reading on Cooperative Learning 249

Appendix C: Assessment and Evaluation Ideas for *The Power of Numbers* 251
Performance Assessment 252
Portfolio Assessment 253
Presentations 254
Student Self-Assessment 254
Investigations 254
Open-Ended Questions 255
Evaluations 256
Selected Bibliography for Further Reading on Assessment and Evaluation 257

Appendix D: Fitting *The Power of Numbers* into Your Curriculum 259

Appendix E: Be Prepared with Special Materials 265

References

Student Handouts

Part 1 Counting on Opinions

A Poll about Young People
- Student Handout 1-1: Monitoring the Future: A Continuing Study of the Lifestyles and Values of Youth *10*
- Student Handout 1-2: Questions from "Monitoring the Future" *13*

Choosing a Topic
- Student Handout 1-3: Choosing Your Polling Topic *24*

The Art of Questioning
- Student Handout 1-4: Bias in Survey Questions *33*
- Student Handout 1-5: Summary *35*

Nuts and Bolts of Statistics
- Student Handout 1-6: What Is Your Population? *54*
- Student Handout 1-7: Good Samples Are Hard to Find *55*
- Student Handout 1-8: Sampling Experiment *57*
- Student Handout 1-9: Fishcounting Data Sheet *59*
- Student Handout 1-10: The Digits of π, $\sqrt{2}$, and 5/17 *60*
- Student Handout 1-11: Summary of Sampling Methods *62*

Designing and Conducting Your Own Poll
- Student Handout 1-12: Checklist for Completing Your Poll *68*

From Highs and Lows: An Average Tour
- Student Handout 1-13: Mean Average *77*
- Student Handout 1-14: Diversity in School Population *78*
- Student Handout 1-15: The Neighborhood Restaurant *79*
- Student Handout 1-16: The Average Arbitrator *80*

Experimenting with Graphs
- Student Handout 1-17: Line Graphs: U.S. Dependence on Foreign Oil *88*
- Student Handout 1-18: Circle Graphs: Land Use in Brazil *90*
- Student Handout 1-19: Bar Graphs: Census Data on American Indians *92*
- Student Handout 1-20: Picture Graphs: World Population *95*

Interpreting Polls
- Student Handout 1-21: Meaning in Percentages *107*
- Student Handout 1-22: "Portrait of a Generation" *108*
- Student Handout 1-23: "Justice Department Contradicts FBI on Crime Rate" *110*
- Student Handout 1-24: Margin of Error in Sampling Television Watching Time *113*

Part 2 Counting on People

The Census: History and Controversy
- Student Handout 2-1: The U.S. Census: History and Controversy *128*

The Census and Reapportionment
- Table A: Representatives under each Apportionment (1790–1980) *148*
- Table B: 1990 Population by State and Numbers of Representatives *149*
- Student Handout 2-2: Reapportionment of U.S. Representatives *152*
- Student Handout 2-3: Classroom Simulation: The Reapportionment of the Country of Frachat *154*
- Student Handout 2-4: Reapportioning by the Webster Method *157*
- Student Handout 2-5: The Hamilton-Vinton Method *159*
- Student Handout 2-6: The Effects of Undercounting *162*

The Census and Redistricting for the House of Representatives
- Student Handout 2-7: Welcome to the State of Billows *166*
- Massachusetts—Congressional districts, Counties, County Subdivisions (Towns), and Places *171*
- California—Congressional District, Counties, and Special Places *172*
- Oregon—Congressional Districts, Counties, and Special Places *173*

Rates of Change
- Student Handout 2-8: Rates of Change, Part 1 *185*
- Student Handout 2-9: Rates of Change, Part 2 *186*
- Student Handout 2-10: Finding a Formula for Rates of Change *187*
- Student Handout 2-11: Rates and Trends over Time: Years of School Completed *188*

Doubling Time
- Student Handout 2-12: Doubling Time *195*
- Student Handout 2-13: Research Project: Find the Current Doubling Time of the U.S. Population *198*

Cross-Referencing
- Table E: Persons 16 Years and Over in the Labor Force by Sex: 1950–1980 *206*
- Table F: Sex Ratio: 1900–1980 *207*

Creating a Public Rail System for Los Angeles County
- Table G: Journey to Work Data for Los Angeles County *214*
- Table H: Journey to Work Data for Los Angeles County (Condensed from Table G) *218*
- Student Handout 2-14: The L.A. Transportation Project: Making a Preliminary Plan *219*

The L.A. Transportation Project, Option 1: Cost Factors
- Student Handout 2-15: The L.A. Transportation Project: Rodent Control Costs How Much? *223*
- Student Handout 2-16: Estimating Operating Costs *226*

The L.A. Transportation Project, Option 2: Traveling on Time
- Student Handout 2-17: Pacing against Time *234*
- Student Handout 2-18: Graphing Time and Rate *236*
- Student Handout 2-19: Traveling on Time *238*

Acknowledgments

During the several years that we wrote this curriculum, many people have offered ideas, encouragement, support, and teasing (for taking nine years). There are several people and institutions whom we wish especially to thank.

The precursor to this curriculum was a packet of materials created with much effort in the summer of 1984 by Fred Gross, Barbara Lipski, Rachel Poliner, Dan Proctor, and Abbie Schirmer. Some of the ideas raised that summer have flowered in this guide; others will find their place in subsequent curricula.

Following that effort, the group that continued to meet for three years included Shelley Berman, Fred Gross, Patrick Morton, Rachel Poliner, and Dan Proctor. Much of the inspiration and enthusiasm for this project originated in those meetings. The group developed an outline for a book that consisted of four chapters, of which the first appears in this book as Part 1 on polling. Some of the suggestions for later chapters (government statistics, influence, and forecasting) have found their way into specific exercises or discussions in this book. The formula-making theme and the idea of using mathematical tools to study trends were two of the important ideas that came out of these meetings. We also had many stimulating discussions about the nature of mathematics and about mathematics as a language that greatly influenced the current version of this book.

We wrote much of *The Power of Numbers* as it now stands on the beautiful summertime campus of Wellesley College, though we were stashed away in the math department's computer lab. The college's facilities were immensely helpful.

Though we started writing this guide in 1984, it was a grant from the State Street Foundation in 1991 that finally spurred us on to complete the manuscript and to test the material in a local school. A generous grant from the Harold Whitworth Pierce Trust supported a pilot project in seven schools, as well as revisions.

The pilot project was very important in giving us feedback for revisions. We thank those teachers and students from Belmont's summer school, Boston's Fenway Middle College High School, Cambridge's Tobin School, Carlisle Middle School, Greenfield Middle School, Hull High School, Medford High School, New Perspectives School, and Newburyport's R.A.N. Middle School who used portions of this curriculum and gave us their thoughts. In particular, we thank Melanie Dana for her revision of Student Handout 1-8 and Georgann Mirick and Cathy Nelson for Student Handout 1-9. We also thank participants in our 1992 Summer Institute for their suggestions, corrections, and feedback.

There are a number of people who helped us find specific information for activities. The Boston office of the U.S. Census Bureau responded to our numerous calls with articles and data. The staffs of Representative Barney Frank and Representative Joe Kennedy sent us helpful articles on reapportionment and redistricting.

Massachusetts Bay Transportation Authority staff members Peter Scholnik, John Powers, Alan Castaline, and Art Conley all helped us estimate costs for the Los Angeles

transit project. Metro-Dade (Florida) Transit Agency staff members Andrew Miller and Guillermo Rodriguez supplied us with transit data from their system as well. We appreciate their time, enthusiasm, and flexibility and their help in putting all the figures in a form that would work for our purposes.

We wish to thank Clancy Wolf for a lesson on spreadsheets and Joan Ross for introducing us to Fermi Questions, which influenced the activity "Estimating Enumerators." We also thank Fred Goodman, whose questions and stories directly influenced our chapter on elections (which will have to be postponed to another curriculum) and whose ideas are subtle influences in many of our activities. We hope we have offered more games than puzzles.

Elizabeth Pritchard and Bethany Chaney helped us find data to complete the curriculum. Elizabeth and Sally Orme solicited permissions for reprints. Sharon Bassett helped with the preparation of the document disks. Sonja Latimore and Ada Beth Cutler offered *many* important suggestions.

To all of these people and their institutions, we offer many thanks.

Introduction

We started to write *The Power of Numbers* in 1984. At that time we and other teachers in Educators for Social Responsibility were creating the *Participation* series of curricula, designed to help students learn about active and constructive citizenship. While working on *The Power of Numbers*, innumerable people asked us how a mathematics curriculum could be part of that mission. It seemed clear to us. We saw that many students were hesitant to learn about issues or develop their own opinions because they could not understand the mathematical and scientific aspects of issues about which they were concerned. We thought, how can educators help prepare young people to understand our world without enabling them to communicate about and through mathematics? Mathematics is part of the language of public discourse in the world in which we live. Students must have power with this language in order to participate fully as citizens.

Imagine a math class in which students are expressing these thoughts:

"I have to understand math so I can make decisions."

"Teach me to do _____. I need to know it for this problem."

"I did it a different way. Let me show you."

Imagine a math class in which the students are energetically speaking the language of mathematics to each other and not waiting for the teacher to tell them everything, where there is active debate about which techniques to use and which processes to try and what the results mean. In such a class mathematics would be *alive*.

This book provides mathematical experiences in real-world contexts that help students interpret, experiment, communicate, and look for multiple solutions to complex problems. This allows students to take center stage. It sets up situations in which the students are the questioners, solvers, and sources of their own learning and in which the teacher becomes observer, guide, and learner. Hopefully, it will help students to experience the power and joy of mathematics and to learn to see mathematics as a fascinating part of the world they live in.

How many mathematics teachers have heard the following comments from their students?

"What do I need to know this for?"

"When will I ever use this after the test?"

"Oh, I've done that already. I just don't remember."

"Is this the right answer?"

Taken seriously, these questions reflect the students' learning experiences in mathematics as well as their real desire to understand the meaning behind mathematical skills and concepts.

To address the problems in the current teaching of mathematics, a variety of new materials are needed. This curriculum offers one piece of what is needed for middle-school and beginning high-school math classes, a curriculum that uses mathematics in context to help students develop higher-order thinking and communication skills. We think that it can also serve as a model for teachers to find other contexts through which to teach mathematics.

Why Mathematics in Context?

Teaching mathematics in context can serve many purposes. This approach encourages students to interact with each other and the material. It promotes curiosity and inquisitiveness. Students begin to see that skills and concepts are part of a connected body of knowledge. The context gives students a reason to learn and remember mathematical skills and shows them how these skills and concepts are applied in actual practice. It also shows them that they can use mathematics for learning about the world.

The National Council of Teachers of Mathematics states: "Students should have numerous and varied experiences related to the cultural, historical, and scientific evolution of mathematics so that they can appreciate the role of mathematics in the development of our contemporary society and explore relationships among mathematics and the disciplines it serves: the physical and life sciences, the social sciences, and the humanities."[1]

This book provides a variety of thematic contexts for mathematical skills and mathematical understanding, including polling, studying trends in census data, and designing a public rail transportation system for Los Angeles. Through these activities, students participate in discussions, interpret and analyze data, make decisions, and present their ideas.

In the context of thematic applications, students see how otherwise apparently unrelated skills are related. They can learn about the interdisciplinary nature of real problems. For example, in the Los Angeles County rail transportation project, students represent data on a map, recognize patterns, design a transit system within a budget, calculate with large numbers, convert units, experiment with the distance-rate-time formula, and work with linear equations.

Context also helps to model uses of mathematics. Students will see that people in many different jobs need mathematics to solve problems and make decisions. This is clear in many activities, including the Los Angeles project and the activities on redistricting for the House of Representatives.

---

[1]National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics* (Reston, Va.: NCTM Commission on Standards for School Mathematics, 1989): 5.

Thinking and Communication Skills

The Power of Numbers encourages students to learn and use many different thinking skills. They will justify, interpret, extrapolate, problem solve, estimate, compare, generalize, and develop intuitive understandings through kinesthetic experiences. There are opportunities to introduce the concept of proof, as students have to provide reasons and arguments for their choices, decisions, and results. For example, in the beginning of the census chapter, students are asked to give a reasoned estimate for the number of people it takes to perform a census. Instead of merely guessing at the number, they must provide a step-by-step explanation of their estimate, including the assumptions they made in their computations. We have also designed activities in which students experience mathematical discovery through several formula-making exercises.

Steven Willoughby states: "Simple symbol manipulation can be done effectively by machines, but higher-order thinking skills and the ability to communicate intelligently about mathematical situations are still uniquely human skills. . . . That does not mean we no longer need the lower-order skills. . . . But people of the future will need more, better, and different mathematical skills. They must be able to relate their mathematical skills to the world around them and use their skills to help solve problems that are of importance to themselves and to the world."[2]

Developing higher-order communication and thinking skills takes practice and new approaches to learning and teaching. Many of the activities, therefore, involve reading, writing, listening, and speaking. It may surprise students to be given reading assignments in math class, like those in the introduction and interpretation section of Part 1 or the background reading on the United States Census at the beginning of Part 2. Through the readings they learn how mathematical questions arise and how mathematical results are reported. We encourage you to have students keep a vocabulary list throughout the curriculum. This will help them use and appreciate mathematics as a language.

Many activities are designed to be done by small groups, in which students have both the on-going need to communicate and an informal medium for practice. Students gain a deeper understanding of material when they have to write about it, explain it to their group members, or prepare to present results to the class. They gain other perspectives on the material when they listen to their peers' approaches. In the averaging activities, for instance, students have to decide which averaging technique to use, explain their rationale to their classmates, and listen to alternative approaches. Ultimately, they can come to understand their own unique style of learning.

---

[2]Steven Willoughby, *Mathematics Education for a Changing World* (Alexandria, Va.: Association for Supervision and Curriculum Development, 1990): 3–4.

How to Use This Book

We hope that teachers and students will lead the curriculum in their classes. Neither this text nor any other should be an inflexible structure leading learning. We encourage you to use these materials in conjunction with other materials and texts. Numerous mathematics textbooks have useful exercises for teaching specific operations and skills that are important for students. We have not included specific skills lessons in this volume because they already exist in so many forms elsewhere. We have, however, listed the skills involved in each chapter on the introductory page for that chapter and in "Math Skills by Section," which follows this introduction. Refer to this if you are looking for chapters that involve particular math skills.

The Power of Numbers can be used in mathematics or social studies classes, or for interdisciplinary study in math and social studies. Students engage in activities that different teachers may be more or less familiar with: calculating and discussing, determining trends and social effects, tabulating results, and making decisions.

The activities in this book do not necessarily have to be used in order. However, there are certain attitudes and mental habits that are important for students to build, especially if you intend for them to do the Los Angeles activity at the end of Part 2. Among these are the stamina to sustain attention on a focused project over several weeks, the flexibility to revise assumptions and answers, and tolerance for uncertainty and multiple perspectives.

Each part of this teacher's guide is divided into several topical chapters. Each chapter starts with an overview and a list of mathematical concepts, skills, vocabulary, and materials involved in the activity (or activities) in that chapter. Each activity includes (just under its title) a list of the handouts, if any, that are involved in that activity. Accompanying handouts are at the end of each chapter. Handouts for students are marked as such. "Going Further" activities are available for students interested in more difficult challenges. Time estimates for actvities are based on class periods of 45-50 minutes. Actual time, of course, will vary depending on your students' needs and interests. Many of the teachers who have used this curriculum reported that longer time blocks encouraged more in-depth treatment of the materials and better group process among students.

Don't Miss the Appendices

There are several appendices that include information, suggestions, and techniques that will be helpful throughout *The Power of Numbers*.

Appendix A correlates each of the activities in this curriculum with the standards for improving mathematics education outlined by the National Council of Teachers of Mathematics.

Appendix B offers suggestions to teachers for cooperative learning techniques that will help students learn cooperation skills and complete activities and projects throughout *The Power of Numbers*.

Appendix C includes many ideas for assessing and evaluating student work, ranging from portfolios to performance assessments to investigations.

Appendix D offers ways to fit *The Power of Numbers* into your schedule if you have only a month or two in which to use the curriculum.

While most activities require only standard school supplies, a few require more unusual items. These are listed in appendix E.

Math Skills by Section

The following sections in this book offer opportunities for students to learn or review the skills listed. We encourage you to use other texts in conjunction with *The Power of Numbers* for teaching specific skills and operations.

Part 1

A Poll about Young People: percentages

Nuts and Bolts of Statistics: population, sample, randomness, proportions, rational and irrational numbers, π, sequences, random sequences, sampling

From Highs to Lows: An Average Tour: mean, median, mode, frequency distributions, stem-and-leaf plots, estimating

Experimenting with Graphs: graphs, area, population density

Interpreting Polls: calculating and interpreting percentages, margin of error, linear equations

Part 2

Classroom Census: percentages, graphing

The Census: History and Controversy: estimating, logical reasoning, working with large numbers

The Census and Reapportionment: percentages, proportions, rounding, working with integers and decimals, exponents, geometric means

The Census and Redistricting for the House of Representatives: estimating area, population density

Rates of Change: rates of change (slope), negative numbers, understanding formulas, graphing

Doubling Time: growth rates, exponential growth, rates of change

Cross-Referencing Census Data: cross-referencing, proportions, linear equations, rates of change

Creating a Public Rail Transportation System for Los Angeles County and The L.A. Transportation Project, Option 1: Cost Factors (Option 2: Traveling on Time): recognizing patterns, logical reasoning, percentages, simple linear motion (rt=d), linear equations, budgets

Part 1 Counting on Opinions

Introduction

Polls are one of the most prevalent tools that governments, advertisers, politicians, businesses, consumer groups, and other organizations use to find out what people think and feel about important issues. Because their use is so widespread, and because they involve many subtle mathematical techniques, polls offer an excellent opportunity to introduce students to beginning statistics and data analysis and to the effects of mathematics on social topics. The context provided by polls helps, in turn, to enliven the mathematics.

"Counting on Opinions" includes data on a variety of social issues as context for learning particular mathematical skills. The social studies skills of detecting bias and understanding influence are also stressed. The larger social studies context, however, is that students learn to use many skills to conduct their own community study.

Part 1, "Counting on Opinions," introduces students to the statistical concepts of population, sample, randomness, averaging, and frequency distribution. There are also exercises on the different uses of graphs, because graphing is one of the ways students can express results and because graphs are used to influence all of us. There are several opportunities to practice the art of estimating. In addition, there are opportunities for students to learn how to write clear questions and do experiments with data. Students will learn about bias in language and in statistics. The goal is for them to learn the main ideas that pollsters use in conducting polls and reporting their results, so that the students can: (1) learn to think critically about information that is presented in the form of poll results and understand what the results of a poll may or may not mean; (2) design and conduct their own polls and use simple mathematical tools to give some meaning to their results; and (3) learn to display their results in a clear and useful form.

An important aspect of Part 1 is that very early on students are asked to choose a polling topic (see "Choosing a Topic"). Students should be given the opportunity to apply the concepts of each chapter in Part 1, *as they learn them*, to the design of their own polls. Thus one of the main themes of Part 1 is that the students can actively create their own context for the mathematics that they learn.

Among the unusual features of the lessons in Part 1 are:

- the use of real numbers, rationals, irrationals, and the number π (pi) for understanding randomness
- the inclusion of several issues: cigarette and alcohol use among teenagers, gun control and ownership rights, deforestation, global population, crime rates, and others
- experiments to understand sampling, bias, and margin of error

- activities through which students learn about bias in language and in choice of mathematical techniques

- extended exercises on graphs, so that students can begin to see why different types of graphs are used in different situations, what their strengths and weaknesses are, and how they can be influential

- exercises to explore the use of different mathematical techniques, especially averaging

- active exercises on estimating, in which students have to construct reasoned estimates

- the sustained opportunity to explore a particular topic, culminating in a large written report and oral presentation.

There are many opportunities in Part 1 to go off on tangents, which can be very fruitful. We do, however, strongly encourage you to move steadily through the early chapters so that you sustain momentum and give ample time to the later activities.

Part 1 begins with a look at the results of the annual survey "Monitoring the Future: A Continuing Study of the Lifestyles and Values of Young People," conducted each year since 1975 by the Institute for Social Research at the University of Michigan. Students read some of the published findings of the survey, in order to get a first look at an interesting poll and to understand the kinds of questions they can ask about polls.

The second chapter of Part 1 contains suggestions and ideas for class discussions on the topics "What Does This Have to Do with Math?" and "Who Needs Math Skills in Social Studies?" Students often come to this curriculum with strong expectations from traditional mathematics and social studies classes. It will be helpful to hold these or similar discussions early in the curriculum to help students gain a broader perspective.

Part 1 has been designed so that, as they go through the material, students gain the tools to conduct their own poll. In line with this, students are asked to choose polling topics very early in the chapter, in Handout 1-3. The chapter entitled "The Art of Questioning" gives students practice in recognizing bias in questions and in developing clear and unbiased questions for the poll they will conduct.

The next chapter, "Nuts and Bolts of Statistics," introduces students to beginning statistical concepts. Included are questions and hints for discussion on populations, sampling techniques, and bias in sampling; a sampling experiment in which students play with variation in repeated samples; another sampling activity related to counting fish in a lake; and a discussion of randomness. The idea of a random sample is connected with the mathematical concept of a random sequence, which students consider by looking at the digits of pi and other irrational numbers like $\sqrt{2}$. This chapter requires that students have some practice with number systems: real numbers, rational and irrational numbers, and their decimal representations.

The chapter entitled "Designing and Conducting Your Own Poll" gives students additional information about ways to conduct a poll. At this point students are ready to

conduct their own poll. The rest of Part 1 focuses on mathematical and graphing techniques for displaying and understanding the results of their poll.

"From Highs to Lows: An Average Tour" includes several activities for distinguishing between the mean, median, and mode and for understanding the uses of frequency distributions and stem-and-leaf plots. The chapter begins with a class exercise on estimating.

In "Experimenting with Graphs" students see interesting examples of line, circle, bar, and picture graphs and compare the strengths and weaknesses of each. Through the picture graph activities, they calculate the area of rectangles to estimate population and create a new graph using their own calculations on population density. They practice correlating information from different graphs. They then return to the poll they conducted and use averaging and graphing methods to describe and display their results.

The graphs with which students experiment are about oil imports to the United States, deforestation in Brazil, the educational and financial status of American Indians and global population distribution. These topics show ways in which information about social issues can be represented mathematically, as well as ways in which mathematical descriptions can be used to influence the issues themselves.

Finally, "Interpreting Polls" suggests ways to look at the results of polls. Coming back to the ideas introduced in the discussion on the "Monitoring the Future" poll, this chapter emphasizes the need for critical evaluation of polls—how they are conducted and how their results are reported. It includes data on the attitudes of baby boomers and a controversy on measuring crime. The section also includes a discussion and exercise on the concept of margin of error. At the conclusion, students write final reports on their polling projects.

The ideas in Part 1, particularly randomness and bias, may seem fuzzy notions at first glance. They can be understood, however, and with practice students can gain facility in recognizing their occurrence and importance. Moreover, the kind of mathematics that students learn in Part 1 gives them a chance to play and create "mathematically" in ways that are denied to them by more traditional curricula.

Many of the ideas in Part 1 recur, in a slightly different and deeper form, in Part 2. The exercises on estimating and comparing graphs, as well as the discussion of line graphs, are precursors to the activities "The Census: History and Controversy," "Cross-Referencing Census Data," and "Rates of Change." Percentages are used throughout both parts and the activity on fishcounting in "Nuts and Bolts of Statistics" uses proportions, which students use frequently in Part 2.

A Poll about Young People

Overview

To introduce polling to students, this lesson and Student Handout 1-1 include quotes and a graph from a national survey about young people. By studying an interesting poll, students begin to think critically about how polls are conducted. At the end of this activity students begin to devise topics for their own polling projects.

Mathematical Concepts

percentages

Skills

asking questions, looking at information critically

Vocabulary

poll, proportion, percent

Materials

copies of Handouts 1-1 and 1-2

Time Needed

two class periods

Activity 1: A Study of Young People, "Monitoring the Future"

Use Student Handout 1-1

The goals of this opening activity are for students to develop an interest in polls and to begin considering the steps that go into polling. They will see that information is collected both about and from young people. After they read and discuss the handout, students should start thinking of topics for their own polling projects.

Have students read Handout 1-1. It lists a few of the results and conclusions of the poll "Monitoring the Future: A Continuing Study of the Lifestyles and Values of Youth." Below you will find more background on the poll. Do not give students this information until they ask for it.

After students have read the handout, encourage them to imagine how this survey was done.

Suggested questions:

- Who did this research?
- How did researchers get this information?
- What questions do you think they asked? Students could try to write all the questions they think necessary to elicit the information for such conclusions.
- Who did they ask?
- How many people did they ask?
- Do these results seem true to you?
- How do the researchers know that these conclusions describe the whole country?
- What mathematical skills were used in the polling process? Students will easily notice the references to percentages and proportions. Encourage them to realize that measuring trends over time requires mathematics and that they will soon encounter other mathematical concepts and techniques.

Students may not be able to answer these questions. The goal of this discussion is to ask the questions, not necessarily to answer them. Help students discover that a lot goes into constructing and conducting a poll and that a lot can be learned from the process. Encourage them to ask many questions and not to accept the results of polls (or other kinds of information presented by the media and advertisers) without looking at them with a critical eye.

Background on "Monitoring the Future"

Since 1975 this poll has been conducted each year for the National Institute on Drug Abuse by the Institute for Social Research at the University of Michigan. Questionnaires are filled out by approximately 16,000 high school seniors from 125 to 135 public and private schools, selected to represent a cross section of the U.S. population. After they graduate, 2,400 of those students fill out follow-up questionnaires in alternating years. Although we listed only a few of the conclusions about smoking and drinking trends, the 1991 questionnaire asked 208 questions about all sorts of things: drug use, self-esteem, age, gender, success in school, feelings of safety, expectations for the future, religion, family background, and much more.

The three groups mentioned in the fifth quote, and the main target populations for the study, are high school seniors, young adults aged 19–31, and college students. (The third group is therefore part of the second group.)

The word "cohort" in the ninth quote refers to someone born in the same year (a birth cohort) or someone in the same graduation class (a class cohort).

As students complete the lessons in this book, you may want to encourage them to do further research into this interesting poll. More information is available through the Institute for Social Research, University of Michigan, Ann Arbor, Michigan 48109 at (313) 764-8271.

Activity 2: A Short Poll from "Monitoring the Future"

Use Student Handout 1-2

Handout 1-2 contains several questions from the "Monitoring the Future" poll, which lets students experience a poll right away and know what's in store.

Give each student a copy of Handout 1-2 and allow time for them to fill out the short questionnaire. Then tell the class to tabulate the results. Help them ask questions about how they might perform this tabulating, but let them come up with ideas themselves. If they have exhausted all their ideas, ask how they could give a picture of the class responses as a whole.

For example, are most students in the class satisfied with the way they get along with their parents (question 6g)? How will they interpret "most"? What percentage of the class answered 5 or higher on this question? Overall, does the class feel pretty good about things? Will single statistics give a good answer to this question, or would it be better to represent some of the information in one or several graphs?

This activity can give students a taste of the kinds of questions that arise throughout the chapter. How can poll results be reported? What role does mathematics play in polling? What kind of average or what kind of graph is appropriate for this data?

Student Handout 1-1

Monitoring the Future: A Continuing Study of the Lifestyles and Values of Youth

After the poll "Monitoring the Future: A Continuing Study of the Lifestyles and Values of Youth" was done in 1989, researchers reported the following results:

> Some 33% of [high school] seniors have had five or more drinks in a row at least once in the prior two weeks, and such behavior tends to increase among young adults one to four years past high school. The prevalence of such behavior among male college students reaches 50% (p. 14).
>
> . . . Experience with alcohol is almost universal . . . (91% of seniors have tried it) and active use is widespread (p. 11).
>
> Some 29% of seniors have smoked cigarettes in the month prior to the survey and 19% already are daily smokers. In addition, many of the lighter smokers will convert to heavy smoking after high school. For example, nearly one in every four young adults aged 19 to 30 are daily smokers (23%), and almost one in five (18%) smoke a half-pack-a-day or more (p. 14).
>
> . . . With the important exceptions of cigarettes and alcohol use, rather little illicit drug use is initiated by sixth grade, according to seniors. (. . . Some 20% already had initiated cigarette use and 10% alcohol use by sixth grade.) (p. 9).
>
> Regarding sex differences in the three populations, males are more likely to use most illicit drugs, and the differences tend to be largest at the higher frequency levels [more frequent use]. Daily marijuana use among high school seniors in 1989, for example, is reported by 4.1% of males vs. 1.5% of females (p. 10).
>
> There remains a quite substantial sex difference among high school seniors in the prevalence of occasions of heavy drinking (25% for females vs. 41% for males in 1989), but this difference has been diminishing very gradually since the study began over a decade ago (p. 12).
>
> In 1989, males and females have just about equal probabilities of being high smokers in each of these populations (p. 13).

Student Handout 1-1 (continued)

While the daily smoking rate for seniors did drop considerably between 1977 and 1981 (from 29% to 20%), it has dropped very little in the eight years since (by another 1.4%), despite the appreciable downturn which has occurred in most other forms of drug use (including alcohol) during the period. And despite all the adverse publicity and restrictive legislation addressed to the subject in the 1980s, the proportion of seniors who perceive "great risk" to the user of suffering physical (or other) harm from pack-a-day smoking has risen only 4% since 1980 (to 67% in 1989). That means that nearly a third of seniors still do not feel there is a great risk associated with smoking (p. 12).

Initiation of daily smoking most often occurs in grades 6 through 9, . . . with rather little further initiation after high school (although a number of light smokers make the transition to heavy smoking in the first two years after high school). Analyses presented in this volume and elsewhere have shown that cigarette smoking shows a clear "cohort effect." That is, if a class (or birth) cohort establishes an unusually high rate of smoking at an early age relative to other cohorts, it is likely to remain high throughout the life cycle (p. 12).

Even experimenting with marijuana is now "out" with most seniors' friends (64%); and a substantial majority think their friends would disapprove if they smoked marijuana regularly (85%). About three-quarters of all seniors think they would face peer disapproval if they smoked a pack or more of cigarettes daily (74%) (p. 149).

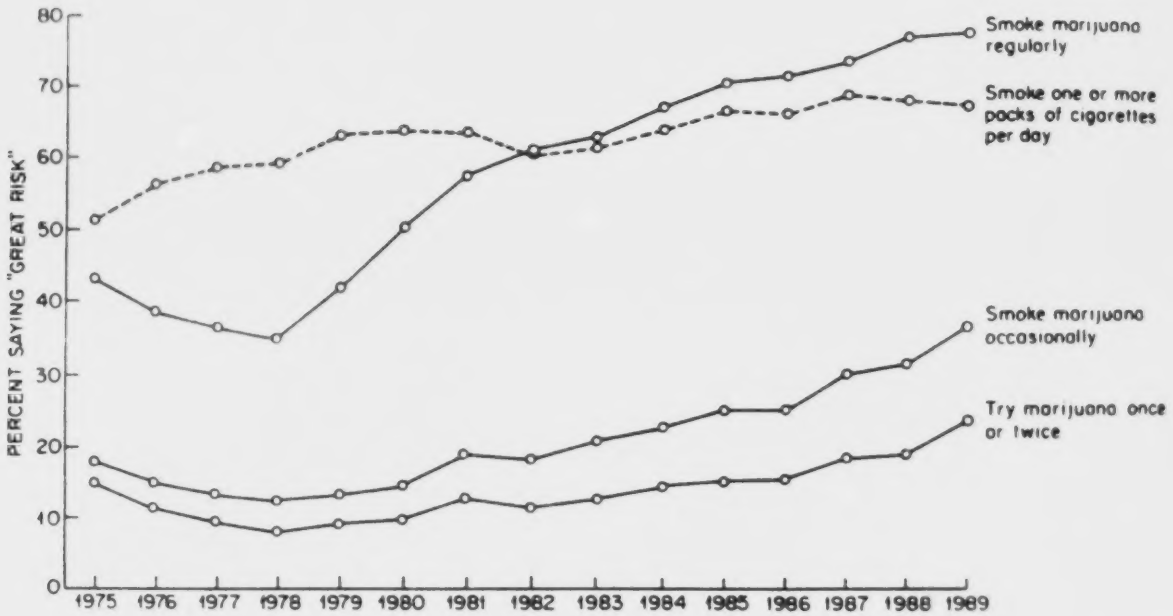
Over 97% of seniors said that their parents would disapprove or strongly disapprove of their smoking marijuana regularly, trying . . . [other drugs], or having four or five drinks every day. . . . (p.147) Also likely to be perceived as rating high parental disapproval (around 92% disapproval) were occasional marijuana use, taking one or two drinks nearly every day, and pack-a-day cigarette smoking. (pp. 147, 149).

As we reported in the 1986 volume, . . . some 53% of the half-pack-a-day (or more) smokers in senior year said that they had tried to quit smoking and found they could not. Of those who were daily smokers in high school, nearly three-quarters were daily smokers 7 to 9 years later (based on the 1985 survey), despite the fact that in high school only 5% of them thought they would "definitely" be smoking 5 years hence. Clearly, the smoking habit is established at an early age and difficult to break for those young people who have it (p. 13).

Student Handout 1-1 (continued)

Trends in Perceived Harmfulness: Marijuana and Cigarettes

All Seniors



Student Handout 1-2

Questions from Monitoring the Future

1. **Taking all things together, how would you say things are these days—would you say you're very happy, pretty happy, or not too happy these days?**

   ◯ Very happy
   ◯ Pretty happy
   ◯ Not too happy

2. **How much do you agree or disagree with each of the following statements?** (Mark one circle for each line.)

	Disagree	Mostly Disagree	Neither	Mostly Agree	Agree
a. The nation needs much more long-range planning and coordination to be prepared for the future	①	②	③	④	⑤
b. I enjoy the fast pace and changes of today's world	①	②	③	④	⑤
c. Things change too quickly in today's world	①	②	③	④	⑤
d. I think the times ahead for me will be tougher and less fun than things are now	①	②	③	④	⑤

3. **Of all the time you spend with other people, about how much is spent with people over 30?**

   ① Very little
   ② Some
   ③ About half
   ④ Most
   ⑤ Nearly all

4. **Would you like to spend more time, or less time, with people over 30 if you could?**

   ① Much less time
   ② Somewhat less time
   ③ About the same as now
   ④ Somewhat more time
   ⑤ Much more time

5. **Would you like to spend more time, or less time, working with or helping younger children?**

   ① Much less time
   ② Somewhat less time
   ③ About the same as now
   ④ Somewhat more time
   ⑤ Much more time

6. **The next questions ask how satisfied or dissatisfied you are with several aspects of your life. For each question, mark the circle that shows best how you feel. If you are neutral about something, or are just as satisfied as you are dissatisfied, mark the middle answer.** (Mark one circle for each line.)

How satisfied are you with . . .	Completely Satisfied			Neutral			Completely Dissatisfied
a. Your job? (If you have no job, leave blank.)	⑦	⑥	⑤	④	③	②	①
b. The neighborhood where you live?	⑦	⑥	⑤	④	③	②	①
c. Your personal safety in your neighborhood, on your job, and in your school—safety from being attacked and injured in some way?	⑦	⑥	⑤	④	③	②	①
d. The safety of things you own from being stolen or destroyed in your neighborhood, on your job, and in your school?	⑦	⑥	⑤	④	③	②	①
e. Your educational experiences?	⑦	⑥	⑤	④	③	②	①
f. Your friends and other people you spend time with?	⑦	⑥	⑤	④	③	②	①
g. The way you get along with your parents?	⑦	⑥	⑤	④	③	②	①
h. Yourself?	⑦	⑥	⑤	④	③	②	①
i. Your standard of living—the things you have like housing, car, furniture, recreation, and the like?	⑦	⑥	⑤	④	③	②	①
j. The amount of time you have for doing things you want to do?	⑦	⑥	⑤	④	③	②	①
k. The way you spend your leisure time—recreation, relaxation, and so on?	⑦	⑥	⑤	④	③	②	①
l. Your life as a whole these days?	⑦	⑥	⑤	④	③	②	①
m. The way our national government is operating?	⑦	⑥	⑤	④	③	②	①
n. The amount of fun you are having?	⑦	⑥	⑤	④	③	②	①

Printed in U.S.A. National Information Services (NIS) 1/91 MP85454.321

What Does This Have to Do with Math? or Who Needs Math Skills in Social Studies?

Overview

At this point many of your students may express discomfort with or uncertainty about how the "Monitoring the Future" activity is relevant to mathematics or why math skills are part of understanding social studies. This is the time to begin a discussion about the nature of mathematics and social studies. Use either or both of the discussions below, depending on the needs of your class.

Rarely are students asked to define mathematics. They may have a very narrow concept of this discipline, dwelling on special topics (algebra, geometry) or numbers and skills (adding, etc.). They tend to have little concept of mathematics as a descriptive tool or as an applied discipline. This discussion can help them understand why mathematics is important and interesting.

Similarly, students often learn history in a very passive way. They may have little understanding of a historian's impact on recorded history or on the variety of available perspectives. They may not realize that social scientists study the past and the present, and also sometimes make hypotheses about the future. This discussion may help them realize that quantitative information can illuminate aspects of history, current issues, or future trends.

Vocabulary

mathematics, comparisons, proportions, percentages, trends, rates, graphs, statistics

Time Needed

one-two class periods

Activity 1: What Is Math?

The point of this discussion is to help students expand their concepts of mathematics. Almost any discussion will start this process, since students' ideas about the nature of mathematics are almost always very rigid.

Ideally, the discussion will occur spontaneously. If students make statements like "This doesn't seem like math to me" or "I don't see why we're doing this in math class" or "This isn't like any other math class I've been in," ask them to explain why. If they are having trouble articulating their thoughts, ask them questions like:

- What do you think mathematics is?
- Is mathematics supposed to be useful? If so, what does mathematics help us to do?
- Why are you supposed to learn mathematics?
- Why does math exist? How did it get created?

Mathematical concepts are most often created to measure something we want to know about, to see how something is changing over time, or to describe a situation in numerical or precise terms. (Much of advanced mathematics is more about relationships between concepts than about numbers per se.) As an example, the students might be interested to know that the word *geometry* comes from two Greek words, geo and metre and means "measuring the earth."

The point of the questions above is to get the students to focus on mathematics more broadly and to take them out of the mindset that sees mathematics only as a collection of independent computational skills.

Another way of furthering the discussion is to ask the students to name some mathematical concepts that appeared in Handout 1-1. Some of these concepts are:

- percentages and proportions ("1 in 4 young adults aged 19 to 30 are daily smokers")
- comparisons ("males are more likely to use illicit drugs")
- trends over time ("the daily smoking rate for seniors did drop considerably between 1977 and 1981")
- rates ("the daily smoking rate")
- graphs
- interpreting statistics and graphs

Now you might ask "Why are each of these things mathematics?" As part of this question you might talk about what each of the concepts means. For example, what is a rate? A rate measures how much something happens in a given time period. Ask them to name some other rates that are used frequently. Or you might ask, "Why is it important to measure trends?" or "How do we measure trends?"

A third way to encourage this discussion is to draw a comparison to another subject area in school. For example, you could ask your students what people should study if they want to be very capable with language. While students will probably say they would need to learn grammar and vocabulary, they will likely also mention the importance of learning to read and write. Ask them to list purposes for these skills. What does language ability enable us to do? Help them see that these abilities enable us to create and understand various kinds of material: essays, poetry, fiction, jokes, recipes, road signs, and much more. They may even see that language skills include speaking and listening. Then return to your questions about mathematics. Does mathematics consist only of computational skills and definitions (grammar and vocabulary)? Or is it also a means for creating and understanding lots of different material? Encourage them to list materials that involve math skills.

Hopefully, students will begin to see that mathematics is a tool human beings have developed for describing and predicting and that this is useful when we are faced with making decisions in the real world. Mathematics is a language, and like other languages it is used to communicate ideas, situations, details, trends, and patterns. It is a specialized language because it needs to be more precise than language ordinarily is; that is why many fields that depend on precision use mathematics.

Students may need to complete a good part of this curriculum (or any other nontraditional curriculum) before they really understand that there is much more to mathematics than they might have originally thought.

Activity 2: What Does Math Have to Do with Social Studies?

If you are using *The Power of Numbers* in what is, or was, primarily a social studies or history class, students may be asking why they are using math skills in this class. It is important to have a discussion showing that mathematical skills are, in fact, useful in studying geography, history, economics, political science, and other fields of social studies. Polling offers particularly rich opportunities to see the blatant and subtle connections between mathematics and society, and discussing these connections at this early point helps students understand the social implications evident throughout this book.

Below are suggestions for:

Activity 2-A: a discussion of and activity on the use of mathematics in social studies

Activity 2-B: a discussion of the effects of polling on social processes, particularly elections

Activity 2-C: an interview activity on the effects of polling on democracy

Activity 2-A: Who Needs Math in Social Studies?

Questions that may encourage this discussion are:

- Pretend that you are going to write a history of your town or neighborhood. What information would you need to do a thorough job? Would you need any numerical information?

- Refer to the "Monitoring the Future" reading. What mathematical skills and concepts did those researchers use? (See Activity 1 for concepts.) Could they have described the same habits of young people without mathematics? Why did they include mathematical information?

- What information do historians needs to write histories? Are there different kinds of histories?

- What current issues interest you? Do any of them involve mathematical information? (Do any of them *not* involve mathematical information?)

With any of the above approaches, help students understand that describing past and current events or situations can involve mathematical skills and information.

For example, if they were to write a history of their town, would they want to use data on people (ages, incomes, races, etc.); measurements of geographical size and location; accurate descriptions of buildings, mountains, or rivers; or descriptions of changes over time? Students will be able to list many items in their histories that would include demographics, area, latitudes and longitudes, trends, and much more.

Ask them if the history one of them would write would be the same as that written by any other student or by an adult or by someone who is not from the area. Who decides what trend is worth measuring? Who decides which events are crucial to include in a history and which ones can be left out?

If you focus on current issues, have students look through newspapers for articles on topics that interest them. Have them list the numerical ideas in their articles and list the questions they have about their topic that might require more mathematical data. If students focus on topics in such fields as the environment, economics, politics (especially in election years), and health, among others, it will be hard for them to find issues that *don't* involve mathematics.

Activity 2-B: Polling as a Mathematical and Social Process

Then, more specifically, ask students how polling, which is a mathematical activity, is involved in current issues and events. This is a discussion to which you might want to return o in different forms at various points in Part 1.

Ask students what effects polls might have on a large scale. For example, do voters change their preferences after they have heard the results of a poll because they want to vote for a "winner"? Or, for instance, are they freer to vote for the candidate they really want (who is not ahead in the polls) because they see that the candidate they like the least

is already out of the running (according to the polls)? Do polls allow citizens' preferences to be known at any given moment, forcing politicians to be responsive to their constituents?

On its "Weekend Edition" program on May 16, 1992, National Public Radio aired a story narrated by Andy Bowers that addressed the question: "Do you feel public opinion polls help democracy or hurt democracy?" Students may be interested in some of the opinions offered by people interviewed. Below are a few of the many responses aired in the story. You and your students can determine which responses propose that polling helps democracy, that polling hurts democracy, or that polling is a mixed blessing.

"Any process that brings leaders closer to public aspirations and public feelings is a positive force in society" (Andy Kohut, pollster).

"We're not really talking about the important issues in any kind of depth at all. . . . What they [polls] do, in fact, is validate minimal compliance when it comes to thinking" (Wes Jackson, Kansas agronomist).

"Polling has been a vital link in the political chain reaction that determines who can raise money, who gets media coverage and who's said to be gaining or losing momentum" (Andy Bowers, NPR reporter).

"Polling won its credibility by adhering to the rules of science, although some critics charge that the strict framework of science can't really be imposed on something as ephemeral as a human emotion" (Andy Bowers, NPR reporter).

"There's always the danger when you as a reporter are talking to people in the street that the people that you talk to aren't necessarily the center of the universe, that they have peculiar or weird views. . . . What a poll helps you do is test a lot more thoroughly and, arguably, more scientifically what the basic attitudes of people in this country are" (Les Rainie, *US News and World Report*).

"People read the polls [which] say foreign policy doesn't matter. . . . People are then asked: do you think foreign policy is an issue in this election? They tend, having been told that it's been found that it is not, to say no. Therefore, no questions are asked of any of the candidates about foreign or defense policy. This is a dumbing-down process where everyone acts as if what everyone else thinks was the determining question" (Christopher Hitchens, *Harper's Magazine*).

"Many critics complain that West Coast voters in 1980 stopped voting after early exit polls showed Ronald Reagan beating Jimmy Carter" (Andy Bowers, NPR reporter).

"Exit polls provide invaluable information about who voted for whom in each election, while still preserving the secrecy of each ballot" (Andy Bowers, NPR reporter).

"The demand is not from the public that we be polled. The demand is from the candidates and special interests that the public be predictable. That's the dream of all demagogues; it's the dream of all political manipulators" (Christopher Hitchens, *Harper's Magazine*).

"This is a democracy. Everyone does count equally. And that's the reason you take polls. If you didn't do polling, only the loudest voices would be heard" (William Schneider, CNN public opinion analyst).

Activity 2-C: Opinions on Opinion Polls

To help bring out many of the interesting aspects of this topic, you may want to have students ask their parents, other relatives, neighbors, or teachers what they think are the effects of polling on our political processes. You could use the opinions that students collect as part of the discussion suggested above.

Choosing a Topic

Overview

Students often choose a polling topic that is familiar to them. It is crucial that students think carefully about what they want to discover, and why. Question them about the importance of their topics, and allow them sufficient time to consider several ideas before submitting a final topic. This assignment is designed to help students clarify and justify their ideas.

Materials

a copy of Handout 1-3 for each student or small group

Time Needed

one class period plus homework

Activity: Deciding on a Topic

You may wish to have students conduct polls individually or in small groups. Have them (individually or in small groups) complete Handout 1-3, in which they pick a polling topic and answer questions about it. Ask them to write their responses in clear sentences. Their responses should be at least two paragraphs in length. They can do this in class or as homework.

Some of the most successful topics may be those that offer a service to the school or larger community. Perhaps the students want to assess their community's need or desire for a park, preschool programs, elder services, or other possible services. Perhaps they wish to assess their peers' attitudes about health, their study habits, or their awareness of important issues. Students can collect and analyze data needed to make a problem known, help make a decision, or understand their own community more deeply.

To help your students select a polling topic, hold a brainstorming session with the class as a whole, in small groups that then report to the whole class, or in pairs. Conduct this brainstorming as a way of introducing the search for topics or after the homework assignment to help students generate further ideas. Some students may need additional individual attention.

It is important for two reasons that students select topics for their polls now. First, developing their topics early will keep their interest high. Second, developing questions, considering populations, and selecting samples—the tasks that follow this activity—take time to do well. When students choose their topics earlier rather than later, they have time to carefully complete these tasks.

As students progress through each chapter, have them apply the concepts and skills of the chapter to their own polls. For example, in the chapter on questioning they should write the questions for their own polls, which gives them an immediate application for the concepts they just learned. Teaching in context requires making that context a continual presence.

It is important for students to select a topic that will hold their interest for several weeks. Therefore, the teacher's input at this juncture is crucial.

Here is an example from a class that used this curriculum. One student wanted to conduct a poll on the extra components students add to their bicycles. After some prodding, the classroom teacher discovered that the student's interest was really in the type of bicycle safety equipment used by different age groups. The teacher's questions helped the student clarify his ideas. Many students will need this type of assistance from the teacher during their initial brainstorming and after the homework assignment. One note of caution: Do not discount the students' ideas too quickly. Rather, gently question them about what they really want to learn and measure.

Here is a sample of student-generated topics:

- student opinions on cigarette smoking
- amount of television viewing
- students' feelings of peer pressure

- bicycle safety features
- residents' opinions on local highway construction
- drugs and alcohol in teenagers' lives
- awareness of environmental issues (rainforest depletion, waste disposal)

Some students wanted to compare the responses from different types of samples. Some examples were:

- How did adults feel about a certain issue as compared to young teenagers?
- How did the responses of eighth-grade and sixth-grade students differ on a chosen topic?
- How did the responses of two different schools compare?

Student Handout 1-3

Choosing Your Polling Topic

What are you curious about that involves people's opinions or habits? A poll might help you find out more.

Your assignment is to complete the following

1. Choose a topic that is interesting to you, because you really want to find out the opinions or habits of others.

2. As a way of justifying your choice of topic, respond, in writing, to the following questions.

   - Why is your poll topic important to you?
   - What do you know about your topic?
   - What do you want to learn from your poll?
   - Will the information be interesting to others?

The Art of Questioning

Overview

Developing carefully worded questions is crucial to conducting a well-conceived poll. This chapter includes examples and activities about the art of questioning that you may want to use in your class discussions. These will help students design questions for their polling projects.

Skills

constructing questions

Vocabulary

bias, open- and close-ended questions, intensity

Materials

copies of Handouts 1-4 and 1-5

Time Needed

three–five class periods

Note

As you go through the following activities, have your students keep a list of terms, types of questions, and response options and have them add new terms they encounter. You may also want to ask colleagues to collect surveys or opinion polls they come across.

The following pages include suggestions for class discussion and student activities. Following the description of activities is a summary for students of things they need to consider when deciding what types of questions to use in their own polls.

Activity 1: Class Discussion on Biased and Unbiased Questions

The first concept to get across to students is the concept of *bias* in questions. The goal of the discussion is to have students understand that

- different questions prompt different answers
- questions can limit responses
- biased questions limit responses in inappropriate ways
- some types of questions encourage answers that may be difficult to record

Example 1

Write the three questions below on the chalkboard. Have the class pretend they just went to a concert and ask them to hold in mind an image of the performance. Ask each of the following three questions and record the students' responses.

1. What did you think about the show?
2. Wasn't that a great concert?
3. Can you believe how short that concert was?

Now compare the results. Question 1 probably prompted longer answers, since questions 2 and 3 can be answered with a simple "yes" or "no." In questions 2 and 3 the opinion of the person asking the question is made clear by the way the question is asked. Question 2 encourages agreement that, yes, the concert was great. Question 3 encourages a complaint that the concert was too short (depending on the inflection in the questioner's voice).

Discuss what effects the different questions had on responses. The following questions may help in this discussion:

- What words in these questions led you to answer in certain ways?
- What opinions might you have had about the concert that questions 2 and 3 did not allow you to offer?

Encourage students to distinguish between question 1, which does allow any response to be given, and questions 2 and 3, which are more specific and biased. Stress the point that if a question is asked in certain ways, it may *prevent* certain answers. For instance, if someone asked you question 3 and you felt strongly that the music was too loud, the room was too crowded, but the dancing was great, you might not feel encouraged to state your full opinion.

Example 2

Write the next set of three questions on the chalkboard. Each question asks about a person's opinion of the weather.

4. What do you think of the weather today?

5. Isn't this cool weather energizing?

6. Don't you hate the weather when it starts to get cold?

Ask the students what difference they see among these questions. Can they guess the opinion of the person who is asking the question?

Explain to students that questions 5 and 6 are *biased* questions. A question is biased if the way the question is asked encourages certain responses and discourages others. *A biased question does not objectively measure opinion.*

Example 3

Here is a third example of a biased question. Write the question and list of possible responses on the chalkboard.

When do you favor mandatory use of seatbelts?

a. some of the time
b. only in urban settings
c. only in rural settings
d. only in areas with populations over 10,000
e. all of the time

Notice that while the initial question calls for any response, only a few responses are listed. Ask the students to name possible that responses are not offered. These include, but are not limited to

- if children less than ten years old are in the car
- for drivers with less than five years driving experience
- in cars, but not in buses or trucks

Explain to students that it is important when asking questions with a choice of responses to allow for all possible responses and alternate points of view. The above question is biased because it does not allow for a full range of responses. One way to avoid such bias is to include options like "none of the above" or "don't understand the question" or "other _____."

Activity 2: Bias in Context

Use Student Handout 1-4

Exercise 1

Give students copies of Handout 1-4. Tell them to examine the questions on the handout. Lead a class discussion using the following questions.

- Which of the questions appear to be biased or unbiased? Why?
- What particular words or suggested consequences in the questions try to sway you?
- Can you tell from the questions what the questioner believes? (This is a signal of bias.)
- Are there questions, which, taken together, point to a certain opinion?

Encourage students to look at phrases that try to scare them or that employ subtle labeling.

Exercise 2

Choose a topic that you think will be interesting to your students, and ask them to write three biased questions that could be part of a questionnaire on this topic. They should be prepared to explain why the questions are biased. Now have the students rewrite these three questions to eliminate bias.

Exercise 3

Ask students if they notice any patterns in biased questions. Examples of phrases they may notice are "If you knew . . .," and "law-abiding." These phrases make you identify with the hypothetical situation described in the question.

Activity 3: The Mechanics of Questions

Use Student Handout 1-2

In the last two activities students considered bias in questions by looking at questions whose wording prompts certain answers. Another consideration in developing questions for a poll is deciding whether to create open- or close-ended questions and whether to allow for intensity of response. This discussion includes information about different types of questions that can be used in a poll.

Handout 1-2 is included in the section "A Poll about Young People." It is from the "Monitoring the Future" poll and shows several types of questions. Use these as examples during this activity.

Start the discussion by offering the following definitions of open- and close-ended questions.

Definitions

Open-ended questions do not require specific answers. The questions are usually short and to the point. For example: "What do you think of the way the president of the United States is doing his job?" or "What are your favorite ice cream flavors?"

Different people generally have very different answers to open-ended questions. They may answer in a few words or have long stories to tell. The advantage of open-ended questions is that they elicit a wide variety of answers that reflect the complexity of the question. The disadvantage is that it can be very difficult and time-consuming to record or compare the responses.

Close-ended questions offer a limited number of possible responses. For example: "Would you say that the president of the United States is doing a (1) good job, (2) mediocre job, or (3) bad job?"

Close-ended questions are useful because responses are easy to record and compare. Most polls use close-ended questions to collect information. Closed-ended questions have the disadvantage of tending to oversimplify issues. If you were asked the above question about the president, you might not feel comfortable with the list of responses you were given. You might think the president is doing a good job in some areas and not in others, but you are required by the question to give an answer that is less complex than what you have in mind.

Some Examples

To investigate these definitions further, go through the following scenarios with the students. Pose the scenario and then ask what type of question, open or closed, would be most appropriate in that scenario.

Example 1. Early in a political campaign, pollsters may want to know all the prospective candidates who are popular with voters. Later they'll be interested only in voters' preferences among the candidates who will actually appear on the ballot.

Ask the students what types of questions they would use in these two situations. In the first case an open-ended question would be useful, such as "Who would you like to see run for president?" In the second case a close-ended question would be appropriate, such as "Will you vote for Candidate A or Candidate B?"

Example 2. Let's say that the only store near your school carries two kinds of juice, apple and grape, and that you have to buy juice for the class. You want to know as precisely as possible how many of your classmates prefer each flavor. Think for a minute about what kind of question you would ask.

A close-ended question would yield the necessary information: "Do you want apple juice or grape juice?" With an open-ended question such as "What kind of juice do you want?" your classmates could list all sorts of flavors that are not available at the store. That wouldn't have helped you at all.

Now let's say you are opening a store for the class and want to know *all* the different flavors that anyone would buy. What kind of question would you ask?

Here an open-ended question would be appropriate: "What kinds of juice do you like?" You may decide later to stock only the most popular flavors, but it would be helpful to know the whole range of preferences of the class. If you ask a close-ended question like "Does everyone like apple juice?" you would receive answers of either "yes, I like apple juice" or "no, I don't like apple juice". If nobody likes apple juice, you have no information about what to stock.

Exercise

Have the class create six questions: three open-ended and three close-ended. It takes some practice to understand this difference easily, so it may be useful to do this with the class as a whole, writing the questions on the chalkboard.

You may want small groups of students to create their own questions and have students interview each other. If you prefer this option, split the students up into groups of two. Ask each student to interview her or his partner using six questions: three open-ended and three close-ended. They should record each question and each response.

Questions for a Follow-up Discussion

- What kind of information is better uncovered using open-ended questions? Using closed-ended questions?
- Were the answers to your open-ended questions or to your close-ended questions easier to record? Why?
- Which questions gave more information? Explain how.
- Which technique allowed the responder to really give his or her opinion?
- Which type of question gives information that is easiest to sort into groups or to make generalizations about?

Allowing for Intensity of Opinion

A third type of question allows the responder to record the *intensity* of his or her feeling on an issue. Pollsters often want to know what people think about something *and* how strongly they feel about their opinion.

To use the campaign example again, pollsters may find out that more people prefer Candidate A than Candidate B. However, they could also ask a second question, "Is

this an important election in your opinion?" or "Do you like Candidate A a little more than Candidate B or a lot more?" The pollsters may find that while Candidate A is more popular, supporters do not feel strongly enough to vote in bad weather. This strength of feeling is called intensity.

Example 1. Present the following two questions to your class:

1. Should students be permitted to eat food in class? yes ___ no ___

2. Indicate the degree to which you agree with the statement "Students should be permitted to eat food in class." (1 indicates strong agreement, 3 indicates no opinion one way or the other or a mixed opinion, 5 indicates strong disagreement.)

   1____ 2____ 3____ 4____ 5____

Ask your class both of these questions (or create several similar questions of your own), and record the results.

In discussing these questions, consider the following points:

- How would you describe the differences between the two questions?
- Do the responses to both questions give you the same information?
- Which question gives you data that is easier to work with?

Ask the class to interpret the overall responses from both questions.

Example 2. A state representative has to vote on a bill before the legislature. She wants to know if her constituents agree or disagree with the bill's intent, but she also wants to know how intense their feelings are.

She can have her staff conduct a yes/no poll or a poll offering a range of responses from 1 to 9. Ask the students to pretend that they conducted both types of polls.

- If you are one of her staff members, how would you interpret the following possible scenarios? How do her constituents want the representative to vote? Does she have constituents who will be angry at her?

1. In the yes/no poll, half the responses are "yes" and half are "no."
2. In the 1–9 poll, half the responses are 1 and half are 9.
3. In the 1–9 poll, all the responses are 4, 5, and 6.
4. In the 1–9 poll, 100 responses are 1 and 2, 150 responses are 4, 150 responses are 5, and 100 responses are 6.

- Does one of these methods seem better suited to the information the representative is interested in learning?

- In which of the situations 1–4 might using mathematics help you to interpret the responses? (In the last three questions it might help you find the average response.)

You may want to go further with the last question by asking students how they would find the average response in 2 and 4. This leads to the idea that mathematical techniques can be helpful in interpreting the responses to a poll. More about this later.

Activity 4: Length of the School Year

Use Student Handout 1-5

Students can experiment with questions that measure intensity by conducting a poll of students, faculty, and parents that asks "Are you in favor of lengthening the school year?" They need to offer a list of responses that give the people they ask an opportunity to add strength to their opinions. See the list of "typical responses to close-ended questions" in the summary for students (Handout 1–5).

Activity 5: Writing Questions

Use Student Handout 1-5

Now ask students to write questions about the topic they have chosen. After they have written first drafts of their questions, have them read each other's to see if the questions are unclear or if they can detect bias. Students can then rewrite their questions.

Student Handout 1-4

Bias in Survey Questions

The following questions come from different surveys attempting to gauge public opinion on several bills before Congress relating to the purchase of handguns. Which of these questions appear biased to you?

1. Would you favor or oppose a national law requiring a seven-day waiting period before a handgun could be purchased, in order to determine whether the prospective buyer has been convicted of a crime or is mentally ill?

   Response (from Gallup poll on the Brady bill in 1991): 87% in favor, 10% opposed, 3% no opinion.

2. If you knew that because of inaccuracies in criminal records some law-abiding citizens could be denied the right to purchase a handgun, and under this bill they would have no way to appeal, would you then favor or oppose this waiting-period bill?

   Response (from "Results of a National Telephone Survey of Registered Voters on Waiting Period and Immediate Check Legislation," conducted by Lawrence Research in 1991): 54% in favor, 42% opposed. The bill referred to is the Brady bill.

3. If you knew that if this bill had been law prior to the attempted assassination of President Reagan, it would not have prevented John Hinckley from buying a gun, would you then favor or oppose this waiting-period bill?

   Response (part of same poll as in 2): 40% in favor, 57% opposed. The bill referred to is the Brady bill.

4. There is another bill that is being debated in Congress. This bill would require an immediate criminal-background check, similar to credit-card check or telecheck by telephone, on people wishing to purchase handguns. It would standardize and update criminal records nationally so that an immediate background check could be conducted. Generally speaking, would you favor or oppose this bill?

   Response (also from same poll in 2): 83% in favor, 15% opposed. The bill referred to is the Staggers bill.

5. In general, do you feel that the laws covering the sale of firearms should be made more strict, less strict, or kept as they are now?

   Response (from Gallup survey, March 1989): 70% more, 6% less, 22% keep the same, 2% no opinion.

6. Is there more crime in your area than there was a year ago, or less?

   Response (from Gallup survey, September 1990): 51% more, 17% less, 24% same, 8% no opinion.

7. Is there any area near where you live—that is, within a mile—where you would be afraid to walk alone at night?

   Response (from 1990 Gallup poll): 40% yes, 59% no.

8. Would you favor or oppose the registration of all handguns?

   Response (from 1990 Gallup poll): 81% favor, 17% opposed, 2% no opinion.

9. Would you favor or oppose a law requiring that any person who carries a gun outside his or her home must have a license to do so?

   Response (from 1990 Gallup poll): 81% favor, 17% oppose.

10. Do you have a gun in the house?

    Response (from 1990 Gallup poll): 47% yes, 52% no, 1% no opinion.

Student Handout 1-5

Summary

Checklist for Deciding What Types of Questions to Use

In creating questions for a poll, you must decide what type of questions to ask, based on the kind of information you are interested in learning. For example:

- Will you allow any response to your question or will you allow people to choose from a list of a few responses?
- Will you ask for people's opinions and weigh them all equally?
- Will you allow people to state both their opinion and whether or not the issue in question is important to them (intensity)?
- Do you need to find an average or majority opinion? If so, you'll need to have opinions you're able to compile and compare easily (close-ended questions).
- Do you want to collect lots of different opinions? In that case, you won't want to limit the responses (open-ended questions).

Typical Responses to Close-ended Questions

- often/sometimes/never
- agree strongly/agree/have no opinion/disagree/disagree strongly
- yes/no
- true/false
- none of the above
- on a scale of 1 to 10, with 1 being low and 10 being high

Nuts and Bolts of Statistics

Overview

This chapter introduces students to the elementary statistical concepts necessary for understanding how polls are constructed and reported: populations, samples, and randomness. These concepts are particularly well suited to student experimentation. The lesson includes activities with the number π (in an unusual context), and sampling activities that involve repeated experiments. The sampling activities are necessary for understanding the notion of a "margin of error," which appears in the last section of Part 1. The work with randomness gives students the opportunity to understand randomness in a concrete *mathematical* setting, preparatory to understanding why random samples are useful.

An excellent reference for this material is David S. Moore's book *Statistics, Concepts and Controversies*, 3rd edition (New York: W.H. Freeman and Co., 1991), which gives a readable and nontechnical account of many statistical methods and ideas. Several of our examples and terms come from this source.

Mathematical Concepts

population, sample, randomness, proportions, rational and irrational numbers, π sequences, random sequences, random sampling

Skills

sampling, solving proportions, working with decimals, finding decimals of rational numbers

Vocabulary

population, sample, bias, convenience sample, systematic random sample, stratified sample, areas, random, random sample, multistage random sample, rational, irrational

Materials

copies of Handouts 1-6 through 1-11, decks of cards (two for each group of two or three students), beads or beans of two different colors, or small pieces of paper of two different colors (one hundred or so for each group of two or three students)

Time Needed

ten–twelve class periods, depending on which activities you choose

Activity 1: Populations and Samples

Use Student Handouts 1-6, 1-7

Handouts 1-6 and 1-7 introduce students to the basic concepts of elementary statistics: population and sample. After the students read the handouts, have a discussion that includes the questions on both handouts.

Help the students understand that part of asking good questions in a poll is knowing what *population* you intend to question. In constructing a good poll, students must first focus on the population and the particular characteristics of that population that they want to measure.

Questions on Population (Handout 1-6)

Different answers to the questions on population are possible. Which population you focus on depends on which population really interests you. Some possible answers to these questions are:

1. Everyone who saw two or more movies this year. (Or: everyone who has seen at least one movie this year. Or: everyone who sees movies "fairly regularly.")

2. The students in your school. (Or: the students in your school district, the students in the whole country, or all the high-school students in the whole country.)

3. High-school students, including those who have dropped out.

4. All former smokers.

5. All people who wear contact lenses.

While the goal of a poll is to learn something about a given population, it is often impractical to ask questions of everyone in the population. That is why pollsters take a *sample* and ask their questions only of people in the sample. The sample may be relatively small (a typical sample is 1,000–2,500 people) or very large (the short form of the government census is sent to 83% of the households in the country, while the long form is sent to 17% of the households in the country). The larger the sample, the more accurate the results tend to be.

Samples are sometimes used for surprising purposes. They are used, for example, to determine composers' royalties for songs played on the radio. It is too cumbersome to have each radio station keep track of every song it plays, so ASCAP (an organization of composers) samples radio programs across the country. Song experts then listen to the tapes and count how many times each song was played. The result of the count in this sample is then used to distribute royalty income among composers.[1]

---

[1]Exan ple taken from David S. Moore, *Statistics, Concepts and Controversies*, 3rd edition (New York: W.H. Freeman and Co., 1991), p. 7.

- What is the population in the above example? (All songs played on the radio in the country in a given year.)

A similar example occurs in the airline industry. Since it is too expensive for airlines to keep track of exactly how many riders buy tickets through competing airlines, every month a sample of 12% of the tickets sold in that month by all airlines is taken to determine how much money different airlines owe each other.[2]

- What is the population in the airline example? (All airline tickets bought by consumers in a given month.)

The classroom discussion of samples should include some questions about what constitutes a "good sample."

Questions on Sampling (Handout 1-7)

The following answers to the questions on Handout 1-7 give some of the main points that might be covered in a discussion of sampling. Encourage students to think creatively about the many possible methods they could use in these examples. Furthermore, while there are many good answers and many weak ones, there is no one "right" answer.

Question 1. A sample is a good sample if it reflects the truth about the whole population. A sample is *biased* if it systematically excludes a certain portion of the population or if it includes a disproportionate number of people from special segments of the population.

Each of the samples mentioned on Handout 1-7 is really a biased sample. Make clear to the students through the discussion that they do not yet know any good methods of sampling. The best methods depend on randomness, a concept we will talk about in the next few activities. More than that, it is probably safe to say that *every* sampling method is biased in some way. Most questions have so many variables that it is almost impossible to obtain an unbiased sample. The best we can usually do is reduce as much as possible the most obvious kinds of bias and then try to be aware of the types of bias that might remain.

Make sure the students understand the idea of a *convenience sample.* This is a sample in which you do the easiest thing possible in picking your sample: you let people come to you. The problem with convenience samples is that they almost always exclude certain portions of the population.

For example, in the first sample mentioned in Handout 1-7 (interviewing 200 people at a supermarket), not everyone goes to supermarkets. People also tend to have certain preferred supermarkets, so in a given supermarket you will probably find only people with special preferences or in certain salary brackets. One way to lessen the effect of

---

[2] Ibid., p. 6.

this type of bias would be to take samples at many different types of supermarkets, in different parts of the city.

There might also be a problem with the time of day that you choose to take your sample. Suppose you found all 200 bubble-gum chewers at the supermarket between 9:00 a.m. and noon on a weekday. Since most young people are in school, this sample misses an important part of the bubble-gum-chewing population. One way to correct this might be to go to supermarkets at many different times, including during the afternoon, evening, and weekend.

In the second sample, by asking the first 100 students you see in your own school, you would be missing adults, younger children, and students in grade levels not in your school.

Even the third method of sampling, calling every sixth (or tenth or fiftieth) phone number in the phone book, is biased.

- Why is this method biased? (Students may guess that this method of sampling is biased because not everyone has a phone. Many people have unlisted phone numbers as well.)

The last method of sampling (sampling using the phone book) is an example of *systematic random sampling*. (You should emphasize again that just because it is a systematic random sample does not mean it is necessarily unbiased.)

- How might you take a systematic random sample of the students in your school? (Use a list of the students by alphabetical order, or by homeroom, and contact every twentieth name on the list, or every tenth, depending on how many people you want to include in your sample.)

While you can choose a sample in many ways, it is difficult to find samples that really are representative of the population.

Question 2. Methods of sampling might be:

- Ask every fourth or fifth person in a listing of students by homeroom.
- Pick a selection of homerooms at random and ask all the students in each of the selected homerooms.
- Have everyone in a gym class fill out a short questionnaire.

These are fairly good samples, if your population just consists of the students at your school. A more biased method of sampling would be to ask all the students you can find who hang out at Burger King every afternoon.

If you wanted to sample people in your city, you might use a phone book and a systematic random sample. Another method would be to use a systematic random sample with a list of registered voters. Both of these methods have some built-in bias. A third method might be to take a listing of the communities in your city by zipcode

and visit them, going door to door to interview people. Still other methods might be to ask your local newspaper to include a list of your questions in their regular deliveries. (Why would the last sample be biased? Not everyone carries a subscription to the newspaper.)

Question 3. The sampling method in *a* is not really representative, since people on the honor roll probably tend to study more than other people. The same goes for the method in *b*, since only certain students will bother to pick up the questionnaire and fill it out. You would probably have a very low response rate! The method in *c* is biased only because the students in one homeroom would not give you a very big sample. The more students you can reasonably include in your sample, the better the sample will be.

Another method might be to pick every third homeroom and have everyone in these homerooms fill out your questionnaire.

You could also obtain a listing of students by grade-point average (GPA) and take a different sample from students with different GPAs. For example, you might divide the students into groups by GPA and pick every tenth (or whatever) student in each of these groups. This type of sample would be called a *stratified sample*. Stratified samples are useful when you can identify different subgroups of the population that would probably have different behaviors, interests, or opinions. The word *stratified* comes from *stratum* (pl. *strata*), which means layer.

Activity 2: Sampling Experiment

Use Student Handout 1-8

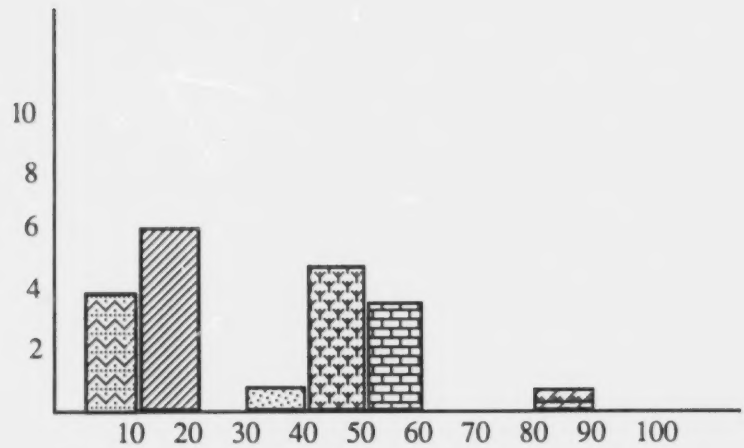
This activity gives students an opportunity to experiment with taking samples in a concrete setting.

For this activity, have students work in pairs. (If the groups are too large, the students will not all be active.) For each pair fill a container with 100 pieces of paper (or beads or beans) of two different colors, say red and black. (If your paper pieces are not red and black or if you are using objects other than paper pieces, substitute your colors and object name in the discussion below.) You could also use stones from a game of *Go* or decks of playing cards. There need not be an equal number of pieces of each color.

Give each pair one container and tell the students to follow the instructions on the handout. Make sure they first take 10 samples with a sample size of ten and then take 10 more samples with a sample size of twenty. After they take each sample, they should put it back into the container and thoroughly mix the pieces before taking the next sample. If students are conducting the experiment with cards, the easiest thing to do is to shuffle the cards thoroughly and either take cards off the top or take every other card to create the sample. Whatever the method, *the same method should be used consistently* for all ten samples of each size.

In step 5 the students are asked to make two bar graphs of their results. The graphs (different graphs for different sample sizes) should show the number of samples in which red pieces constituted various percentages. For this it is convenient to divide the

horizontal axis into increments of ten (five would work well also): 0–10, 11–20, 21–30, etc. See the example below.



In the example, there are four samples in which 0–10% of the pieces are red, six samples in which 11–20% of the pieces are red, five samples in which 41–50% of the pieces are red, etc. Note that this graph is not what you might expect from such an experiment, but it is a possible distribution of twenty-one different samples. Students will account for a total of ten samples in each of their graphs.

The point of the exercise is for students to see that a sample will not have exactly the same characteristics as the whole population. (In this case, the characteristic being measured is the percentage of red pieces.) However, the sample percentages will tend to cluster around the percentage in the population. Also, the larger the sample size (relative to the population size), the closer the sample percentages will cluster around the population percentage.

This makes plausible the idea that one can learn significant characteristics of a population by focusing on fairly small samples. Not any sample will do, however. The method the students are using in this activity is called *random sampling*. Random samples are special because their characteristics do cluster around the population characteristics. There is more discussion of randomness and random sampling in Activity 3.

Variations

There are many ways to adapt this experiment to show the effects of bias in a sampling method. Students can change the size of the population and/or the sample or alter the shape, size, and color of the pieces.

Variation 1. Have students experiment with pieces of different sizes. Provide containers filled with a mixture of one-inch and two-inch squares of cardboard. Have

the students repeat the steps on the handout, but ask them to predict the results of the experiment beforehand. They will likely see the two-inch squares showing up in their samples four times more often then they exist in the population.

Variation 2. Then have the students continue with two sizes, now in two colors: half large, half small, half red, and half black. They can count the percentages of each of the four types of pieces. For the bar graph, ask them to graph the percentages for each size separately. (In the activity as described on the handout, they are asked to graph percentages for only one color.)

- How do the bar graphs for different colors compare?

- How do the bar graphs for different sizes compare?

They may find that graphs for the different colors are similar (small black similar to small red, large black similar to large red), but that graphs for different sizes will be different (small black should show smaller percentages than large black). (These remarks will be true only if each of the four types are equally represented in the population.) This variation shows how populations can have different attributes.

Variation 3. Have the students experiment with pieces of different shapes (squares, parallelograms, circles—shapes whose areas they can easily measure) and keep track of the percentage of each shape occurring in their samples. Students could also use yogurt lids and plastic milk jug lids, which work well in this experiment. For this exercise you will probably need to have a larger population in order to get interesting results.

First have students do the experiment using shapes with different areas to see if the shapes with larger areas tend to occur more often in the samples. Then have them try different shapes with the same areas.

- Which shapes seem to show up more often in the samples they pick?

Encourage students to experiment with different combinations.

Going Further with Sampling: Fishcounting

Use Student Handout 1–9

The following activity gives the students another opportunity to experiment with samples and sample size. In this activity students learn how to estimate the number of fish in a lake by sampling and solving simple proportions. Give them Handout 1-9 for the purpose of record keeping. Before they begin you will need to conduct the following discussion with them.

Sampling is sometimes used for environmental purposes, as in estimating the number of fish in a lake. Counting the fish in a lake is done in several stages.

1. The first step is to take a *tagging sample.* This is just a sample of fish, all of which are tagged and then put back into the lake.

2. The second step is to take a *counting sample.* This is a second sample of fish that is caught and counted. When the sample is taken, some of the fish that are caught will be the fish tagged in step 1 and some will not. Counting the number of tagged and untagged fish will help us estimate the total number of fish in the lake.

We now make an assumption: that the proportion of tagged fish in the second sample is the same as the proportion of tagged fish in the entire lake. As an equation,

$$\frac{\text{tagged fish in 2nd sample}}{\text{total fish in 2nd sample}} = \frac{\text{no. of tagged fish in lake}}{\text{total no. of fish in lake}}$$

- What makes this a reasonable assumption? Are there problems with this assumption?

At this point we know three of the quantities in this equation; namely, the number of tagged fish in the lake, the number of tagged fish in the second sample, and the total number of fish in the second sample. By solving the proportion for the quantity we do not know, we can find (approximately) the total number of fish in the lake.

Example. Suppose our tagging sample contains 200 fish and our counting sample contains 300 fish, of which 50 are tagged. An equation for the proportion of tagged and untagged fish is

$$50/300 = 200/x,$$

where x represents the total number of fish in the lake.

- Now solve the equation for x. What do you find? (x = 1200.) This gives us an estimate for the total number of fish in the lake.

Discussion Questions

1. How should the counting sample be chosen so that the above equation more accurately represents the truth? For example, would you want to take the second sample immediately after putting the tagged fish back into the lake? Or in the same part of the lake? (An analogy here might be helpful: sampling again without allowing the fish to mix would be like picking the cards off the top of a deck of cards without having shuffled well. You will get back the cards you just put there.)

2. Can you imagine situations in which the percentage of tagged fish in the sample might not be close to the percentage of tagged fish in the whole lake? What could you do to get a more accurate estimate?

One way to approach the last question is to conduct the following experiment.

Activity on Fishcounting

The process used for counting fish can be simulated using ordinary playing cards or a box of pieces of paper as in Handout 1-7. Start with several decks of playing cards shuffled together. Use decks with different backs so the students can tell cards of the same type apart. (For example, the two of hearts from one deck must be distinguishable from the two of hearts from other decks.)

In this experiment the cards represent the "lake" and a certain subset of the cards represents the "fish." For example, the "fish" could be represented by the red cards, by the diamonds, by the black cards with striped backs or by the face cards.

Students can conduct the simulation using the following steps. For clarity, suppose we let the red cards represent the fish.

Step 1. Take a tagging sample by shuffling the cards well and turning up the top thirty cards. Tag the "fish" in this sample by recording the tagged cards; for example, list all the red cards in this sample:

the two of hearts
the seven of diamonds
the queen of diamonds, etc.

Now put these thirty cards back into the deck. Record the total number of tagged "fish."

Step 2. After shuffling thoroughly, take a second sample of cards, say forty. This is the counting sample. Count the number of cards that are tagged, that is, contained on your list of tagged "fish" from the first sample, and the total number of "fish" in this sample. Record these numbers.

Step 3. Use the numbers from step 2 to compute the percentage of tagged fish in the counting sample. Record this number.

Step 4. Use the information to estimate the number of "fish" in your lake.

- Did you run into any problems? What could you do to avoid these problems?

Step 5. Now repeat steps 2–4 for a new counting sample and record the new data. Repeat this at least ten times. For each sample there is an estimated "fish" population. How do these ten estimates compare to each other and to the

actual "fish" population? Find the average (mean) of these estimates. How does this number compare to the actual population size?

The students may want to set up their data as in the following table. In this table we have assumed that there were twenty "fish" in the tagging sample.

Sample #	Total # of tagged fish	# Fish in sample	# Tagged fish in sample	Estimated fish pop.
1	20	20	4	100
2	20	30	10	60
3	20	17	2	170
.				
.				
10	20	35	5	140

Mean estimate = 117.5

The mean the students obtained in step 5 may not be very close to the actual population size. This mean is affected by many variables: the size of the tagging sample, the size of the counting sample, the way samples are taken. Have the students experiment with tagging samples and counting samples of different sizes, to see how close they can come to the actual population size.

- Does the size of the counting sample affect the accuracy of the estimated number of "fish" in the population? If so, how? Describe the relationship.

- How is the mean estimate affected by taking more counting samples for the same tagging sample? For example, above you were asked to take ten different counting samples. What if you took fifteen or twenty?

- Summarize your results in several paragraphs. Say what you can about the relationship of your individual sample estimates to the actual number of "fish" in the population.

Activity 3: Randomness

Use Student Handout 1-10

This activity introduces the students to the idea of randomness. Before starting this activity, students should understand real, rational, and irrational numbers and should have had some practice turning fractions into repeating decimals and repeating decimals into fractions. These topics are covered in standard texts.

Randomness is difficult to define precisely, so in what follows the goal is to understand some of its characteristics. Studying *random sequences* will help the students get a handle on the real-world problem of random sampling.

Sequences

A *sequence* of numbers is just an ordered list of numbers, like

1, 2, 3, 4, 5, 7 (no misprint); or

1, 1, 3, 5, -1, .5; or

3, 3, 3, 3, . . .; or

1, -3, 5, -7, 9, -11, 13, -15, . . . or

1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, . . .

A sequence can be finite or infinite. The first two sequences above are finite, while the last three are infinite. Make sure students understand the patterns in the last three examples. This is a good time to give them more experience in recognizing patterns. Ask them to name some sequences. Make clear there is infinite variation in the way sequences can be written down. Give them other sequences and ask them if they recognize the pattern.

Example 1. Do you recognize how the following numbers are formed?

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, . . .

There are two possible answers: (1) this is just the list of squares of positive integers; (2) you can get each number from the previous one by adding successive odd numbers:

1 + 3 = 4,
4 + 5 = 9,
9 + 7 = 16,
16 + 9 = 25, etc.

Because, there are different ways of seeing this means an interesting formula is lurking in the background. In this case the formula is that the square of the number N equals the sum of all the odd numbers up to 2N - 1.

Example 2. How are the following numbers formed?

2, 3, 10, 15, 26, 35, 50, 63, 82, 99, 122, . . .

It is a good idea to leave the sequence from example 1 on the board when you put this one up. Then students may see that this is the list of squares again, but a 1 has been alternately added to or subtracted from the squares:

1 + 1 = 2
4 - 1 = 3,
9 + 1 = 10,
16 - 1 = 15, etc.

Exercise. Have students invent their own patterns and try them out on each other. Or divide the class into groups and have the groups compete to find patterns of sequences you give them.

Puzzle. Here is a mind-bending example of a sequence (each number in the sequence is written on a different line):

2
12
1112
3112
132112
1113122112
311311222112
...

Ask students if they recognize how each number is constructed from the previous number. This is really a difficult pattern, so give them plenty of time (overnight or even a whole week). The (one?) answer is: the pattern is easier to spot if you count the numbers out loud, digit by digit—the first line has one 2, the second has one 1 one 2, the third has three 1s and one 2 . Each number counts and names the digits in the previous number! The last number is built by calling out the sequence of digits in the previous number: there is one 3 one 1 two 2s two 1s one two. What would the next number be? (13211321322112)

Random Sequences

Now we'll just focus on sequences of *digits*.

A *random sequence of digits* is, roughly speaking, a sequence in which every element is a digit (0, 1, 2, ..., 9) (though you can also think about digits in bases other than base ten) and in which every digit appears roughly the same percentage of the time. But this property is not enough to guarantee randomness. It is actually a little easier to say when a sequence of digits is *not* random.

A sequence of digits is *not* random, if (roughly speaking) some digits occur significantly more than other digits or some combinations of digits occur more often than other combinations, or if there is a recognizable pattern to the digits.

It is easy, of course, to come up with sequences of digits that are not random, such as

1, 2, 4, 1, 2, 4, 1, 2, 4, . . .

1, 3, 7, 8, 1, 2, 1, 3, 7, 8, 1, 2, . . .

These sequences are not random because not every digit appears in them. But the sequence

0, 2, 5, 6, 7, 1, 3, 4, 9, 8, 0, 2, 5, 6, 7, 1, 3, 4, 9, 8, . . .

is also not random because its digits appear in the repeating pattern 0256713498. Knowing the pattern, you can predict what the next digit will be at any point.

Example 1. Ask students whether they think the following sequence is random?

1, 0, 2, 0, 0, 3, 0, 0, 0, 4, 0, 0, 0, 0, 5 (then five zeros), 6 (then six zeros), . . .

(Answer: no, because 0 occurs significantly more than any other digit.)

Thus, in a random sequence, every digit appears with roughly the same frequency (and every thinkable *combination* of digits occurs equally often; see the more precise statement below) and there is no recognizable pattern to the digits. Sequences of random digits are useful *because* you *cannot* predict what the next digit will be.

The most famous example of a sequence of random digits is the sequence of digits of the number π, pi:

π= 3.1415926. . .

Mathematicians have searched for patterns in the digits of π but have never found one. For this reason the digits of π, which have been computed to millions of places, are often used by mathematicians to create random sequences.

Handout 1-10 asks students to a count the number of times each digit appears in the first 201 digits of π, the first 201 digits of $\sqrt{2}$ (an irrational number), and the first 16 digits of 5/17 (a rational number).

Help students understand that for a sequence to be random, every digit must appear in the sequence roughly the same percentage of the time. Each digit should appear about twenty times in each of the first 201 digits of the numbers π and $\sqrt{2}$.

For the numbers π and $\sqrt{2}$, not only does every digit appear roughly 1/10th of the time, but every pair of digits (including repeated pairs like 44, known as "twins") occurs roughly 1/100th of the time, every group of three digits appears together 1/1000th of the time, and so forth. This could be checked experimentally by taking many more digits and counting pairs. Interested students might want to count how many times each pair of digits does occur in the digits listed. (Each pair should occur roughly twice, but some will occur more often and some not at all.)

The final number on the handout, 5/17, does not lead to a random sequence of digits, if the decimal digits are continued, because the digits repeat periodically. Even a calculator will not show this, unless it shows at least 20 digits or so. Note that the digits of 5/17 repeat after every sixteenth digit!

Every rational number either has a repeating or finite decimal expansion, so rational numbers are not useful for creating random sequences. (Note: rational numbers are the numbers we see most often in practice, because we can deal with only a finite number of digits at a time.)

Most irrational numbers have a random sequence of decimal digits, though many irrational numbers do not; for example the digits in the number

.10100100010000010000001...

are certainly not random. But the number is irrational because its digits do not repeat periodically.

Three Experiments on Randomness

1. Ask students to flip a coin fifty times and record whether each toss is heads or tails. Write a 1 each time the coin comes up heads and 0 each time it comes up tails. Does the resulting sequence appear to be random? (Make clear that here we want to check only whether 0 and 1 come up approximately equally often (we are dealing with digits base 2), and that the sequence does not appear to be predictable.)

2. Ask the students to roll a single dice fifty times and record the results. Does the resulting sequence of numbers (all between 1 and 6) seem to be a random sequence? Have the students create a frequency distribution for these fifty trials.

   In both of these experiments, the sequences that students find should appear random. If they do not appear to be random, you have found a really interesting phenomenon!

3. Ask the students to roll a pair of dice fifty times and record the *sum* of the dice. Does the resulting sequence of numbers (between 2 and 12) appear random? (This time the answer will most likely be no, since the number 7 should appear significantly more often than 2.)

Going Further with Randomness

In each part of this activity students are given a specific way of creating a sequence of digits and are then asked if the resulting sequence is random. In the first two examples they should carry the computation far enough (to, say, twenty places) to be able to tell whether the resulting sequences appear to be random or whether they can predict the next digit. In the final exercise students are asked to come up with their own method of creating a random sequence.

1. Start with a digit x. Square it, add 1, and take just the final digit. Repeat this for the new digit. Does this process lead to a random sequence? Experiment with

different starting values for x. (This process leads to a periodic sequence, which should repeat every tenth digit or earlier. It is therefore *not* random.)

2. Start with a digit x between 2 and 9. Cube it, subtract 2, and take the final digit. Repeat the process. Does this give you a random sequence? Experiment with different starting values for x. (Same answer as in 1)

3. Start with x, a number between 0 and 1 (x = .2 say). Create a sequence of real numbers between 0 and 1 in the following way. Subtract x squared from x and multiply the resulting number by 4.

   a. Write a formula for this process.

   b. Now take the result and apply the process again. Repeat fifteen times.

   c. Next, take the list of decimals that you have just created and take the first three digits of each number. Make a new sequence by listing these digits in the order in which you found them. Does the sequence appear to be random?

   d. Experiment with different starting values for x.

(A formula for the process is x goes to $4x - 4x^2$. The sequence may or may not seem random. Depending on which starting values you take, there may be a preponderance of 1s and 9s.)

4. Invent your own method for creating a sequence of digits. Does it give you a random sequence?

Activity 4: Random Samples

Activity 3 is intended to give students an intuitive feeling for the elusive concept of randomness. It may be interesting to follow this with a discussion of randomness in daily life, before trying to apply the idea to sampling.

Ask the question:

- Do you think the word *random* applies to any of the following: the moon's orbit? tides? how you get to school? (Do you take different paths, or is the way you get to school predictable?) the way you decide what to wear in the morning? the food you eat? the times you eat?

It appears that many of us behave habitually. Much of our lives occurs in cycles. This behavior is analogous to the behavior of digit sequences that are periodic. Thus we would not call habitual behavior random.

- Can you think of anything you do that has an aspect of randomness to it?

It might even be interesting for students to watch their behavior in some area over a period of weeks to see how habitual or random what they do is.

Given the human tendency to do things in cycles, it is no wonder that it is difficult to take a sample that reflects the characteristics of a given population. This is why pollsters resort to the subtle mathematical concept of randomness to do their polling. It has been discovered that if a sample is taken *randomly* (in ways we will talk about), then it has a very good chance of representing a population well.

Example. How can you use the concept of randomness to do a phone sample?

One way is to use a random sequence of digits. Starting at the beginning of the sequence, mark off the sequence in sevens. Each group of seven digits then represents a phone number. A sample created in this way is called a *random sample.* Since the sequence of digits is random and is created in a way that gives unpredictable results, the phone numbers are likely to reach a cross-section of people, even those whose phone numbers are unlisted!

- Can you think of any problems with this method? How might you deal with these problems?

One problem might be that the seven-digit numbers created in this way do not correspond to phone numbers in your area or in areas you want to reach. There are several ways to get around this problem. One is to simply ignore the phone numbers with exchanges outside the area you are interested in calling. Another is to first use the sequence of random digits to pick a random set of exchanges in your area and then create random four-digit numbers to be the second parts of the phone numbers. Then you could randomly match the three-digit exchanges to the groups of four digits to find your phone numbers.

Samples that are created using randomness in several stages are called *multistage random samples.* Some samples of this type also involve drawing numbers out of a drum or tossing a coin.

To Summarize

A *random sample* is a sample in which each member of a population has an equal chance of being questioned. (This is analogous to each digit in a random sequence appearing with roughly equal frequency.) When a sample is random, it has a good chance of reflecting the opinions and preferences of the whole population.

The best way to ensure that a sample is truly random, and therefore reflects the population at large (as closely as possible), is to use a method in which chance decides what members of the population are included in the study. In selecting a sample for study, the students must ensure that every member of the population being studied has an equal chance of being selected. If this is not true, then the study is biased in some way.

Exercise

Ask the students to devise a method for taking a random sample of the students in your school.

Activity 5: Pick a Sample

Use Student Handout 1-11

Review with students the different types of sampling methods that have been discussed. Give them Handout 1-11 and ask them to write in an example of each type of sampling and the types of bias that might occur in that example. Or you may choose to give them this handout earlier, so they can take notes on samples as the discussion progresses.

Give students the following writing assignment: what are three ways to select a sample for your polling project? They should explain what they identify as their population, how they could select samples, and the bias involved in each of the three methods.

Ask students to then choose a sampling method that they will use for their project. Help them select a method that is both appropriate for their topic and manageable.

Student Handout 1-6

What is Your Population?

As we have seen, polls are tools for collecting information and are usually conducted by asking a certain group of people a specific set of questions. This sounds easy enough.

But collecting information that is useful and interesting can be a complicated business. After designing questions that will help you find out what you're really interested in knowing, the next step is to decide who to poll.

Population

The *population* is the entire set of individuals or objects about which you are interested in obtaining information.

For example, if you wanted to know which bubble gum is preferred by people who chew gum, the population would consist of *all* bubble-gum chewers, probably a pretty large group.

In the government census the population consists of everyone in the country, since the government is interested in knowing about all of us.

However, if you were interested in knowing what music is preferred by your friends, your population would be just the set of all your friends.

Questions on Population

1. If you want to know what movie is most people's favorite this year, what could your population be?

2. If you want to ask opinions about homework, what could your population be?

3. If you are investigating the high-school dropout rate, what is your population?

4. What is your population if you are investigating the health of smokers who have quit smoking?

5. What is your population if you are investigating contact lens preference?

Student Handout 1-7

Good Samples Are Hard to Find

It is usually much too difficult to poll everyone in the population about which you want to find information. For this reason, pollsters take a sample of the population and poll only the people in this sample. The trick is to find a sample that reflects the views or the preferences of the whole population.

Samples

The *sample* is the part of the population you intend to interview in your poll. The number of people in the sample is called the *sample size*.

Let's say you want to find out which is the preferred bubble gum among gum chewers, but you can't interview all current bubble gum chewers. Then you would find a *sample* of people who chew gum and ask them what bubble gum they prefer.

There are many ways of choosing a sample, though some give more accurate results than others. One way to choose a sample would be to go to a supermarket and ask the first 200 people who buy gum which bubble gum they prefer. Another way to take a sample would be to ask the first 100 students you see in school who are chewing gum which bubble gum they like best. Each of these samples is called a *convenience sample* because you're not doing anything special to pick whom you poll.

A third way to pick a sample would be to call every sixth (or tenth or fiftieth) phone number in the phone book in your area, determine whether anybody is home, ask them if they chew gum, and, only if they say yes, include them in your sample. This kind of sample is called a *systematic random sample*.

Pollsters generally choose a sample for practical reasons. It allows them to find answers to their questions and makes the data from the poll more manageable. The important and difficult task is to pick a sample which really does reflect the conditions, characteristics or opinions in the whole population. Unfortunately, there is no way to guarantee that this is the case. But there are ways to increase the likelihood that a sample will be representative of the population

Questions on Samples

1. Think of two additional ways to find a sample of gum chewers. Which of these methods, or the methods described above, is a good method of finding a sample, in your opinion? What makes a good sample good?

2. Say you are interested in knowing how many people in your school prefer Pepsi-Cola over Coca-Cola. What are some different ways you could choose a sample? Name at least three. What if you wanted to ask the same question about your city or town? What are some reasonable samples?

3. If you're trying to find out the average amount of time that students in your school spend studying, and it is not feasible to question all the students in the school, how appropriate would the following methods of sampling be?

   a. Interview the students on the honor roll.

   b. Leave a questionnaire in a box outside the school office, to be filled out by whoever passes by and wants to fill it out.

   c. Interview the students in one homeroom.

   Can you think of other methods?

Student Handout 1-8

Sampling Experiment

You will be given a container filled with 100 red and black pieces of paper. This represents your population. From this population you will take samples and compare the percentage of red pieces in the samples to the percentage of red pieces in the whole population.

Step 1. Without looking, take ten pieces out of the container. This is your sample. Record the number of red pieces in the sample in the "Sampling Experiment Data Sheet" table. Return all the pieces to the container and mix them thoroughly. Choose another sample and repeat the process until you have taken ten samples.

Step 2. Now increase the number of pieces in your sample to twenty. Take ten different samples, again recording the number of red pieces in each sample.

Step 3. Compute the percentage of red pieces in each of the samples you found in steps 1 and 2.

Step 4. Create two graphs, one for each sample size, with percentages from 0 to 100 by tens on the horizontal axis and the numbers 1 to 10 (for the number of samples) on the vertical axis. Use a bar graph to record the number of samples that fell in each range of ten percent: 0–10%, 11–20%, 21–30%, etc.
- Do you notice any similarities or differences between your two graphs?
- Based on your two graphs, estimate the percentage of red pieces in the population.

Step 5. Count the number of red pieces in your population and express it as a percentage.
- How do the percentages of red pieces in your samples compare to the percentage of red pieces in the population?
- Does the size of the sample have any bearing on the accuracy of your percentages? Explain.
- What is the average percentage of red pieces in these samples? (To find the average, add the percentages from all the samples together and divide by the total number of samples.) How close is the average to the percentage in the population?

Sampling Experiment Data Sheet

Name _____ Date _____
Name _____ Container # _____

Sample size *10*

Sample #	# of red pieces	% of red pieces

Sample size *20*

Sample #	# of red pieces	% of red pieces

Total number of red pieces in the container: _____
Percentage of red pieces in the container: _____

Student Handout 1-9

Fishcounting Data Sheet

Name _____

Name _____

Sample #	Total number of tagged fish	Number of fish in sample	Number of tagged fish in sample	Estimated fish population

Student Handout 1-10

The Digits of $\pi, \sqrt{2}$ and 5/17

1. The first 201 digits of π are

   $\pi = 3.14159\ 26535\ 89793\ 23846\ 26433\ 83279\ 50288\ 41971\ 69399$

   $37510\ 58209\ 74944\ 59230\ 78164\ 06286\ 20899\ 86280\ 34825$

   $34211\ 70679\ 82148\ 08651\ 32823\ 06647\ 09384\ 46095\ 50582$

   $23172\ 53594\ 08128\ 48111\ 74502\ 84102\ 70193\ 85211\ 05559$

   $64462\ 29489\ 54930\ 38196$

   Count the number of times each of the digits 0, 1, 2, . . . (up to 9) occurs in this sequence. Compute the percentage of total occurrences (out of 201) for each digit.

   - What do you find?
   - Do the digits of π seem to form a random sequence?

2. The first 201 digits of $\sqrt{2}$ are

   $\sqrt{2} = 1.41421\ 35623\ 73095\ 04880\ 16887\ 24209\ 69807\ 85696\ 71875$

   $37694\ 80731\ 76679\ 73799\ 07324\ 78462\ 10703\ 88503\ 87534$

   $32764\ 15727\ 35013\ 84623\ 09122\ 97024\ 92483\ 60558\ 50737$

   $21264\ 41214\ 97099\ 93583\ 14132\ 22665\ 92750\ 55927\ 55799$

   $95050\ 11527\ 82060\ 57147$

   Count the number of times each of the digits 0, 1, 2, . . . (up to 9) occurs in this sequence. Compute the percentage of total occurrences (out of 201) for each digit.

- What do you find?
- Do the digits of $\sqrt{2}$ seem to form a random sequence?

3. The first 16 digits of the number 5/17 are

   5/17 = .29411 76470 58823 5

   If you continue this sequence of digits by finding 5/17 to more decimal places, will the digits form a random sequence?

Student Handout 1-11

Summary of Sampling Methods

When taking samples, you have many possible methods to try. The following is a list of some of the kinds of samples we have discussed in class. In the space provided, list at least one example of each method and one type of bias that might occur in your example, if any.

Convenience sample. A sampling method in which you do the easiest thing by polling people you run into in some specific location.

Random sample. A sample in which chance decides what members of the population are selected.

Systematic random sample. A method in which you take a list of the population and systematically pick members off the list by a certain rule: take every fifth person or twelfth person, etc.

Sampling using random digits. A method in which you use random digits to simulate chance. Usually you associate numbers to members of the population (or list them) and use the sequence of random digits to pick the sample.

Multistage random sample. In a multistage sample you do the sampling in several stages. First think of a way to contact your sample in stages. For example, you might divide the population by area (where they live) and then by street. Choose at random several areas and then choose at random streets in each area.

Stratified random sample. This is a sample in which you divide the population into categories and sample in each category separately.

Historical Note[3]

America's first great polling fiasco came not during the 1948 presidential campaign, when so many pollsters predicted that Thomas E. Dewey would beat Harry S. Truman, but during Franklin Delano Roosevelt's 1936 campaign for reelection. In those days, a New York magazine called *Literary Digest* ran the oldest and most highly publicized survey on presidential campaigns. It had picked the winner of presidential races from 1920 through 1932. In 1936, the *Digest* wanted to conduct the broadest possible survey, so its editors . . . settled upon two lists: a compilation of names from telephone directories and a list of people who had registered automobiles.

Months before the election, the *Digest* mailed out 10 million mock ballots to people whose names appeared on the two lists. By late October the *Digest* received 2,376,523 completed ballots. On the basis of that information, the *Digest* predicted that Alfred M. Landon would defeat Roosevelt in a landslide, that the president would receive a mere 41 percent of the vote. . . .

As it turned out, of course, Roosevelt won in a landslide, taking 63 percent of the popular vote and winning every state except Maine and Vermont. . . .

Henry E. Brady, an associate professor of political science at the University of Chicago and an expert on polling, says the *Digest's* flaw was as obvious as it was fatal. "In the midst of the Depression," says Brady, "only the rich had telephones and automobiles." As a result, he says, the *Digest* wound up with a sample in which there were too many wealthy people and Republicans and not enough poor people and Democrats.

Thus did *Literary Digest* commit polling's mortal sin: It did not base its survey on a random sample of the electorate.

---

[3]This excerpt is from Charles Kenny, "They've Got Your Number," *The Boston Globe Magazine*, August 30, 1987, p. 12. Permission to reprint granted by *The Boston Globe*.

Designing and Conducting Your Own Poll

Students have now chosen a topic for their poll, have had practice writing unbiased questions, and have thought about ways to find random samples. They have one more decision to make: How will they actually conduct the poll?

The suggestions and questions below form the basis for a class discussion to help students make decisions about the particular method they will use to conduct their poll.

Class Discussion: Designing a Poll

Start by asking students what methods they could use to conduct the poll, that is, to actually gather responses to the questions they want to ask. After they have decided how to pick a sample of the population they are interested in, how will they contact the people in the sample?

Have students keep their topic and population in mind as they discuss various poll-taking methods. For example, interviewing door to door in the immediate neighborhood may be appropriate if the topic is how people in the neighborhood feel about something. However, if a student wants to know how people in the whole town feel, he or she will want a less time-consuming method.

Here are some methods they may mention. (If they do not suggest them, offer them as alternatives.) They might design a questionnaire for people to fill out in the presence of the poll taker, interview individuals in person, interview individuals over the phone, interview small groups of people in person, or mail out questionnaires to be returned. For each suggestion, ask the students what advantages or disadvantages that particular method might have.

For example, if they give a group of people a questionnaire to be filled out (as might happen in a school classroom), the participants may ask one another how they responded or may not take the survey seriously. Yet this method enables pollsters to gather many responses quickly.

Personal interviews require a space that minimizes interruptions. Also, individual interviewing is very time consuming. However, interviewing does encourage more in-depth responses, and also allows the interviewer the opportunity to ask follow-up questions.

With telephone interviewing there is no guarantee that people are available or willing to respond to the call. It can be challenging to call strangers. However, using the telephone has advantages as well. The interviewer can make sure the respondent understands the question, and can ask follow-up questions. Using a telephone also makes it easier to pick a random sample of participants.

In a group interviewing participants can be influenced by the responses of others. Also, when several people react to a question, it can be difficult to quickly and

accurately write down their answers. However, with this method many responses can be collected more quickly than in individual interviews.

Using the mail requires money for postage, and there is no guarantee that people will respond or respond quickly. However, the respondents are free to answer in any way they feel is appropriate. Sometimes people answer written questionnaires more truthfully than oral questionnaires, because they are not worried about what the interviewer will think of their honest opinions.

The method by which students conduct their polls will influence the types of responses they get and their results. Encourage students to carefully consider each method before deciding which course to follow.

If students select telephone or in-person interviews, you may want to have them role play to practice introducing themselves and their topic. If they explain who they are and why they are doing the poll people will be more receptive to answering their questions. Discuss with the students other techniques that will encourage people to respond: politeness, calling at respectful times of day, not getting upset with people who choose not to respond, etc.

Here are some further questions that students might want to consider before settling on a method for conducting their poll.

- Does the method match your personality?
- Do you feel comfortable or uncomfortable talking to new people, groups of people, or on the telephone?
- Do you have the time to conduct your poll using a particular method?
- Are you able to take very clear and thorough notes when others are talking? Would you rather have your respondents write their own answers?

Activity: Constructing a Poll

Use Student Handout 1-12

If they have not done so already, students should think of a topic for their polls, write some questions they think are appropriate, and test them on their classmates to check for bias or unclear wording. Each student should then decide on a method for conducting the poll and taking a sample. Depending on the topic, they may want to try taking a random sample or contacting the whole population (if the population is small enough). These choices may not be possible or desirable in all cases, but they will lead to more accurate results. No matter what method of sampling students choose, they should think about the ways their samples may be biased.

Give students Handout 1-12, which is a checklist of steps needed to complete the polls. Alternatively, you may wish to have them create their own checklist. A star (*) on the checklist refers to an item that is appropriate for only some polls. For example,

students will not need to determine averages if all questions are open ended or have yes/no answers.

Once students have decided on a polling method, they each need to create an appropriate questionnaire.

When constructing a format for their questionnaire, students must pay careful attention to the spacing of the questions. They should allow plenty of room between questions, between the question and answer, and more space for longer responses if questions are open ended.

Next, have students consider formats for responses to the questions. For close-ended questions, they should create a place for either the participant or interviewer to record responses. This may be a box or line to check, an item to be circled, or other techniques the students may devise.

Finally, voice control is very important during an interview. Students should know to be careful not to influence respondents with a loud or suggestive tone. Preventing bias can also take place during the oral stage of an interview; therefore, students should listen carefully to their own voices.

Here are some other points the students should consider when constructing their polls.

- Is the questionnaire easy to read?
- Are the words on the page legible?
- Do the directions to the questionnaire make sense?
- Is the questionnaire one page or several pages?
- If the questionnaire is several pages, is the order of the pages clear to the respondent?
- Sometimes, questionnaires use both sides of the paper. Is this clear to the respondent?

Once students have constructed a format for their questionnaire, they should have their classmates check it for clarity.

Now they are ready to conduct their polls!

Student Handout 1-12

Checklist for Completing Your Poll

Task	Started/Date	Completed/Date
Topic chosen	( ) _____	( ) _____
Written list of questions	( ) _____	( ) _____
Questions checked for bias	( ) _____	( ) _____
Questionnaire checked for format*	( ) _____	( ) _____
Decision made on how poll will be conducted	( ) _____	( ) _____
Conducting poll	( ) _____	( ) _____
Results tallied and placed on frequency distribution	( ) _____	( ) _____
Averages computed*	( ) _____	( ) _____
Graphs constructed	( ) _____	( ) _____
Written summaries of data collected for each question	( ) _____	( ) _____
Interpretations (adding context for the results)	( ) _____	( ) _____
Argument vs. counterargument*	( ) _____	( ) _____

*If appropriate to your poll.

From Highs to Lows: An Average Tour

Overview

Data is often organized through mathematical procedures that simplify, summarize, and represent the data. This lesson focuses on three different kinds of averages that are used in interpreting data: the mean, median, and mode. Some activities also involve frequency distributions and stem-and-leaf plots. The lesson includes a class discussion and several activites that ask students to choose appropriate averages for making decisions. The goals are to introduce students to the mathematical procedures involved in describing data and to have them understand that different methods can lead to different interpretations.

Mathematical Concepts

mean, median, mode

Skills

estimating, calculating averages, making frequency distributions

Vocabulary

per capita, mean, median, mode, frequency distribution, stem-and-leaf plot

Materials

calculators, copies of Handouts 1-13 through 1-16, several "Help Wanted" sections from newspapers

Time Needed

five–eight class periods

Activity 1: Introduction to Averages

One way to begin is to discuss several types of averages with which the students may already be familiar. The following examples ask students to think about three specific averages: average rainfall, average consumption of milk, and per capita income in the United States.

Example 1

Average Rainfall. Ask students to guess the average yearly rainfall of the following locations:

Baltimore, Maryland	New Orleans, Louisiana
Barrow, Alaska	Phoenix, Arizona
Cinncinnati, Ohio	Portland, Maine
Minneapolis, Minnesota	Portland, Oregon
Mt. Waialeale, Hawaii	Reno, Nevada

After students have had a chance to make their guesses, ask them:

- Why did you make the guesses you did? Explain your thought process.
- What does "average yearly rainfall" mean?

The answer to the last question is, of course, that average yearly rainfall is the average rainfall over a specific number of years. The *1990 World Almanac and Book of Facts* (p. 226) gives the average yearly rainfall of the cities listed above as:

Baltimore, Maryland - 73.30"	New Orleans, Louisiana - 74.40"
Barrow, Alaska - 3.59"	Phoenix, Arizona - 7.21"
Cinncinnati, Ohio - 39.97"	Portland, Maine - 43.62"
Minneapolis, Minnesota - 19.08"	Portland, Oregon - 31.72"
Mt. Waialeale, Hawaii- 460"	Reno, Nevada - 5.3"

These are averages taken over the thirty-year period 1951–80.

- Why would knowing average rainfall be important? (Among other things, averages are used to show trends and patterns. Knowing the average rainfall over a number of years helps to predict weather patterns and monitor changes in the environment.)

Example 2

Per Capita Consumption of Milk. Ask students what they would guess the average yearly consumption of milk to be per person in the United States. This is the type of

average referred to whenever the words *per capita* are used. (Per capita means literally "a head.")

Students can begin to estimate the average yearly consumption of milk per capita by considering their own consumption. They can do this by actually measuring and recording their milk consumption over a day or a week, or they can simply try to recall as accurately as possible how much they usually drink in a day. From either of these they can estimate how much they might drink in a year.

Students' results will likely be in quarts or gallons. If their calculations are in cups or ounces, remind them that eight ounces equal one cup, four cups equal one quart, and four quarts equal one gallon.

Next, have them convert their results to pounds, as milk consumption data are recorded. They can use the conversion equation:

11.5 gallons = 100 lbs. or 1 gallon = 8.7 lbs. (rounded off)

After the students have estimated their individual consumption, have them calculate a class average. They can then consider whether their average will be lower, higher, or similar to the national average.

The recorded national average is approximately 220 lbs. of milk consumed annually per capita. The following figures are also from the *1990 World Almanac and Book of Facts* (p. 121) and give the per capita consumption of whole, lowfat, and skim milk for the years 1986–88.

Year	Whole	Lowfat	Skim
1986	116. lbs.	98.2 lbs.	13.4 lbs.
1987	111.4	100.1	14.0
1988	106.1	101.6	16.2

Ask the students:

- How do you think "per capita consumption of milk" is actually computed? (This figure is computed by dividing the total milk sold in a given year by the total population of the country.)

Example 3

Per Capita Income. Finally, ask the students what they think is the per capita income of the United States and how this number is computed. The *1990 World Almanac* (p. 766) gives the figure $16,444 for the year 1988. This resource also gives the per capita income for other countries, which may be interesting for students to guess, just as they guessed average rainfall in the first part of this activity.

Activity 2: Averages and Frequency Distributions

Each of the averages in Activity 1 is an example of a mean average. Two other types of averages are the median and the mode. Give students the following short descriptions of the mean, median, and mode. Activities 3-5 give them practice in using these terms. For further exercises you may want to consult a standard mathematics textbook.

This activity also includes a short intoduction to frequency distributions and stem-and-leaf plots.

The Mean

The *mean* of a group of numbers is the value arrived at when all the numbers in the group are added up and then divided by the total number of elements in the group. Mean averages can be used with numerical values, such as prices, numerical test scores, batting averages in baseball, ages, etc.

As an example, ask your class to find the average ages of ten women, if their ages are

49, 51, 52, 52, 53, 55, 55, 56, 57, and 58

If you follow the procedure for finding the mean, then you should find the average to be 53.8.

Ask the students to explain why each of the three averages (of rainfall, milk consumption, and income) considered above are all means.

The mean of a group of numbers is most useful when the numbers in the group are more or less close together and without extremes.

The Median

The *median* is the middle number in a series of numbers, *when they are arranged in ascending or descending order*. It is the number that divides the numbers in the group in half. Thus half the numbers are greater than the median, and half are less than it.

For example, the median of the numbers 2, 3, 5, 7, 9 is 5.

Thus the median of an *odd* number of real numbers (arranged in ascending order) is the middle number. However, if you want to know the median of an *even* number of real numbers, there are two middle numbers. In this case the median is the mean average of the two middle numbers.

For example:

- the median of the numbers 2, 3, 5, 7 is 4 (the mean average of 3 and 5)
- the median of the numbers 1, 3, 8, 8, 9 ,9, 10, 11 is 8.5 (the mean average of 8 and 9)
- the median of 4, 5, 5, 5, 7, 8 is 5 (the mean average of 5 and 5)

- Ask the class for the median of the following test scores: 100, 99, 97, 96, 95, 94, and 54. (The median average is 96, because three test scores are above and three are below this number.)

The Mode

The *mode* is the value most often repeated in a set of numbers. If a teacher wants to know the score most often achieved on a test, he or she would use the mode.

For example, consider the following test scores.

> 100, 100, 98, 97, 97, 96, 92, 91, 89, 88, 87, 87, 86, 85, 84, 84, 84, 83, 83, 81, 77, 77, 77, 74, 70, 69, 67, 66, 63, 58, 57, 57, 56, 55, 54.

The number 84 is the most often repeated score, and therefore the mode of the scores is 84.

The Range

Another concept that is sometimes useful is the range of a set of data. The range is simply the distance from the largest number to the smallest number in the set.

To find the range, subtract the smallest number from the largest number in the set. In the above sequence of numbers the range is 46, since 100 - 54 = 46.

Frequency Distributions

Averages help to understand a *central tendency* or the middle of a series or set of numbers, but they do not show the full distribution of the data. A frequency distribution helps display where all the numbers are clustered.

A frequency distribution is a count of the numbers in a set of data by interval or class.

For example, count the scores in the set above that lie in each of the intervals 50–59, 60–69, 70–79, 80–89 and 90–100. The resulting count is called a frequency distribution. In our case:

Intervals	*Count*
50–59	6
60–69	4
70–79	5
80–89	12
90–100	8

Sometimes a frequency distribution is given in the form of a *stem-and-leaf plot*. In this type of display, the data is divided into groups of ten, as before, but the second digit of each piece of data is displayed next to the interval in which it lies. For the above set of data we have the following stem-and-leaf plot:

Interval

5| 456778
6| 3679
7| 04777
8| 134444567789
9| 126778
10| 00

Note: for the stem-and-leaf plot we put 100 into a separate category just to be consistent. The series of numbers 0 4 7 7 7 to the right of the 7 indicates the scores 70, 74, 77, 77, 77.

A stem-and-leaf plot is useful because it shows the number of pieces of data in each interval and also shows exactly what each score is. The frequency distribution, on the other hand, suppresses the actual data in favor of the counts.

Activity 3: Mean Average

Use Student Handout 1-13

This activity is designed to give students practice in calculating the mean average from real salaries.

Students will calculate mean average salaries and ranges for several occupations. With this data they can make observations and comparisons, thereby using mathematics to describe particular situations. The calculations will vary depending on the students' newspaper clippings.

To help your students, begin to collect the "Help Wanted" section from one or more major newpapers well before you start this lesson. Enlist the help of your colleagues and friends in collecting copies.

Have students cut out job advertisements. Remind them to choose those ads that list salaries. Then have them tape, paste, or staple their ads by occupation category onto a piece of cardboard or oaktag. Add or substitute occupations in the activity as desired.

Activity 4: Diversity in School Population

Use Student Handout 1-14

This activity contains two sets of statistical data. The goal of the activity is for students to see that claims made using averages do not always describe the data accurately.

Students should evaluate the claims of the principals in the activity by determining which averages have been provided and by making a frequency distribution. Afte they have made their evaluation, have students explain their reasoning.

One answer to the bulleted question from the handout (i.e., what might this mean about the diversity of other classrooms?) is: if the average over all classes is 35%, and in one of the classes 75% of the students are students of color, then some of the classes have to have fewer than 35% students of color. The answers to numbered questions on the handout are: (1) 141 and 116, respectively; (2) both figures are mean averages; (3) the distribution of Elementary School West is spread over a much wider range than the distribution for Elementary School East.

If students calculate the averages for the number of students of color per class, the figures they should get are:

for Elementary School West, the mean is 7, the median is 5.5,the mode is 5, and the range is 0-16

for Elementary School East, the mean is 5.8, the median is 6, the mode is 6, and the range is 4-8

Activity 5: The Neighborhood Restaurant

Use Student Handout 1-15

This activity is designed to help students compare averaging techniques. Divide students into groups of two or three. Students will calculate the mean, median, mode, and frequency distribution of the neighborhood population. Part of the exercise asks them to make a frequency distribution using a set of unusual intervals. The intervals given on the handout might be useful in creating their restaurant plans.

Questions 1, 2, and 3 ask students to determine the mean, median, and mode averages of the population and then to *explain what the number tells them*. Discuss these explanations with the class as a whole. For example, the median average of the population is twenty-one, which tells students there are just as many people younger than twenty-one as older than twenty-one. Does that help them design their restaurant? Students who think first about what the techniques will tell them may decide that they do not even need to know one or more of the numerical results.

In question 5, help students see the connection between the mathematics in this activity (averaging techniques, arranging data in a frequency distribution, weighing the values of both the individual numbers and the numbers in the intervals) and their final decisions.

If you want students to think more about how data can affect decisions, have them describe their restaurants in more detail. What food would they serve? What music would be playing? What would the restaurant look like? What other special features might they offer?

The numerical answers to handout questions 1–3 are:

1. 33.33 (3433 / 103)
2. 21
3. 16

Activity 6: The Average Arbitrator

Use Student Handout 1-16

In this activity students will consider what makes an average appropriate in a particular context.

As mentioned earlier in the chapter, the mean average can be influenced by one or more numbers in the extreme. You may want to point this out to your students by asking them how the numbers in the activity are clustered, and how the range of the numbers can influence the average or central tendency of the numbers.

If students focus only on calculating averages for the whole company, encourage them to identify subsets of employees that might be appropriate for their task. For example, should their decision be based on all company employees or on the workers only?

The answer to question 1 is that $28,846.15 is the mean average of the salaries of all employees. The answer to question 2 is that $18,000 is the mode average of the salaries of all employees or the mode average of the salaries of the workers only.

When students have made their decisions, have them present their arguments to their classmates. Student decisions may vary as there are sound rationales for several different decisions.

Student Handout 1-13

Mean Average

Collect the "Help Wanted" section from one or more major newspapers. The more sections you can find, the better.

Cut out and collect job ads that mention salaries, and categorize them according to occupation.

Sample Occupations

1. Teachers/educators
2. Computer
   a. engineers
   b. programmers
3. Accountants
4. Engineers
5. Salespersons
6. Medical technicians
7. Nurses
8. Social workers
9. Cooks/chefs
10. Communications/advertising
11. Insurance

Now calculate the *mean average salary* for each occupational area. Compare the results. Here are some questions to consider:

- Which occupation has the highest average salary? Why do you think this is so? (Offer as many different reasons as possible.)
- Which occupation has the lowest average salary? Why is this so? (Again, offer many different reasons.)
- What are the salary ranges for each of the occupations?

Using appropriate salary intervals, make a frequency distribution of the salaries you have found. Locate where the mean is on the distribution.

- What are some important factors that contribute to these average salaries?
- If you had to prioritize the importance of these occupational areas, how would you do it? Give your reasons.

Student Handout 1-14

Diversity in School Population

You are a newspaper reporter researching demographic changes in local schools for an article that you are writing. During your research you visit several schools, meet with principals, and attend a number of classes and events.

Two of the schools, Elementary School East and Elementary School West, have caught your attention. The principals of both schools boast about having diverse student populations. They tell you that Elementary School West has an average of 35% students of color enrolled per class, while Elementary School East has an average of 29% students of color enrolled per class. However, impressions from your visits to these two schools raise some questions: some of the classes and events include a diverse mix of students; others do not. For example, while visiting one of the classrooms in Elementary School West you notice that more than 75% of the students in the class are students of color.

- What might this mean about the diversity of other classrooms?

You ask the principals for more data to back up their claims. Each school has a total of 400 students, and a consistent class size of 20 students. Each principal gives you a chart listing the number of students of color per class.

Elementary School West	**Elementary School East**
(12) (15) (7) (14) (1)	(6) (7) (5) (6) (5)
(3) (3) (4) (14) (6)	(5) (5) (8) (5) (6)
(9) (6) (5) (2) (5)	(7) (6) (6) (4) (6)
(16) (5) (1) (13) (0)	(7) (4) (6) (5) (7)

Evaluate each principal's claim that their school has a diverse student population.

1. How many students of color are there in each school?
2. How were the figures 35% and 29% computed?
3. What do you notice about the enrollments of the schools?
4. Does the claim of either principal seem valid? Why or why not?

Student Handout 1-15

The Neighborhood Restaurant

You are designing a restaurant that will open soon. You want to know the average age of the people in the neighborhood so you can set up the atmosphere of the restaurant, including music, menu, and furniture, to attract lots of people.

The question is, what is the average age of the neighborhood population?

Below are the ages of the people in the neighborhood. Each figure stands for the age of ten people.

2	5	9	14	17	21	35	48	65	68	83
2	5	9	14	17	21	36	49	65	70	85
3	5	10	15	17	25	37	50	65	71	88
3	6	10	15	18	26	40	50	65	72	
3	7	10	16	18	27	41	51	66	73	
4	7	11	16	19	28	45	55	66	74	
4	8	11	16	19	30	46	57	66	75	
4	8	12	16	20	30	46	58	67	78	
4	8	12	16	20	30	47	60	67	79	
5	9	13	16	20	31	47	61	68	81	

Questions

1. What is the *mean* average of the population? What does this number tell you?
2. What is the *median* average of the population? What does this number tell you?
3. What is the *mode* average of the population? What does this number tell you?
4. Create a frequency distribution of the ages using the following special intervals:

0–5	17–24	50–64
6–10	25–34	65–79
11–16	35–49	80–up

(Restaurants are especially interested in understanding the numbers of people in each of these intervals. Can you see why?)

5. Now make a decision about the clientele to which the restaurant should cater. Explain carefully how you made your decision. Which techniques did you use and why?

Student Handout 1-16

The Average Arbitrator

You are the arbitrator with the responsibility of helping the management and union of a manufacturing corporation come to an agreement on salary increases.

The union represents 59 workers and is looking for a raise in salary, but is meetig resistance from the board of directors. The chair of the board states that the average yearly salary for employees within the corporation is $28,846.15. Therefore, the board has proposed a $1,000.00 salary increase for the workers for the next fiscal year. On the other hand, the union leader states that the average yearly salary of employees is $18,000.00, and the union would like a salary increase of $7,000.00 per worker for the next fiscal year.

Position **Number in each position**	**Yearly Salary** **per individual**	**Total Salary** **per position**
1 President	$200,000.00	$ 200,000.00
3 Vice-Presidents	$100,000.00	$ 300,000.00
5 Managers	$ 50,000.00	$ 250,000.00
10 Supervisors	$ 30,000.00	$ 300,000.00
11 Workers	$ 28,000.00	$ 308,000.00
20 Workers	$ 20,000.00	$ 400,000.00
22 Workers	$ 18,000.00	$ 396,000.00
6 Workers	$ 16,000.00	$ 96,000.00
		$2,250,000.00

The union leaders and management representatives have rejected each others' arguments and cannot agree on a proposal. Therefore, as the arbitrator, you must decide on the salary increase. But first, you must answer some questions.

1. Which averaging method was used by the chair of the board?

2. Which averaging method was used by the head of the union?

3. What other statistical information might help you make your decision?

4. Now make your decision and explain it in writing, including your rationales and calculations.

Experimenting with Graphs

Overview

These eight activities introduce students to four types of graphs: line, circle, bar graphs, and picture graphs. Students also learn about scaling graphs and experiment with applications of the four types.

Mathematical Concepts

graphs, area, population density

Skills

graphing, understanding the differences among graph types, understanding scales, determining area, measuring in metric units, calculating density

Vocabulary

line graphs, circle graphs, bar graphs, picture graphs, area, ratio, density, scales

Materials

copies of Handouts 1-17 through 1-20, graph paper, metric rulers, large world map, and calculators; optional are metric grid graph paper, overhead transparencies with metric grid on them, computers, and spreadsheet software

Time Needed

seven–ten class periods

Introducing Graph Types

Have students consider each of the following graphs separately. During activities 1 through 4 help students understand the appropriate applications for each graph type by asking them to list and summarize the strengths and limitations of the graph styles they are working on. In activities 5 through 8 students compare forms of graphs, create new graphs, and experiment with transferring information from one type of graph to another type.

To highlight some of the limitations, students will not be able to answer some of the questions on handouts 1-17 through 1-20 using the graphs they are given. Tell

students at the beginning of each of these graph activities to think critically about each question. They have to determine what each graph does and does not tell them.

The questions in activities 1, 2, 3, and 4 ask students to employ various skills: calculating, estimating, writing, comparing, reading, imagining, and measuring. Encourage them to develop their abilities to communicate mathematical ideas by having them write complete sentences.

Activities 5, 6, 7, and 8 offer opportunities for students to use spreadsheet software. Students will be able to see real-world applications for spreadsheet skills. They will also be able to experiment quite flexibly with the graphs involved in those activities—changing the scale, trying out different graph types, transferring data. In each of these activities challenge students to explain why they chose one graph type over another and how the graph styles compare. While computers can be available to help with manual labor, students need to be clear about their decisions.

Have students work in groups of two to three throughout the activities, which will enable them to discuss their thoughts and to delegate tasks such as measuring, calculating, and recording.

Activity 1: Line Graphs

Use Student Handout 1-17

The first graph students work with is a line graph. It is important for them to realize that the strength of line graphs is in showing change, in this case change over time. They can see trends, increases, and decreases and see approximately when they took place. They do not just see a snapshot of certain points, as in a bar graph.

With rulers they will be able to determine that they can see trends until 1990, even though 1989 is the last year marked (question 3). Encourage them to add more intervals to the graph if they need to (particularly for questions 5 and 6). They will not be able to answer questions 4 and 7. The line graph does not show them the relation between oil use and other or total energy use, as a circle graph would.

Activity 2: Circle Graphs

Use Student Handout 1-18

The circle graph shows the relationship between part(s) and whole. Circle graphs show proportions and relative size or amount very well. They give an overview. This will be apparent when students answer question 8, as they will have a sense of land use throughout the state of Rondônia.

Circle graphs are, however, just a snapshot. By themselves they do not show change over time. Two circle graphs, as in the handout, show change, but they do not show exactly when the change occurred. The changes could have happened all in one short spurt or gradually over the fifteen years. Thus, students will not be able to accurately answer question 3.

Question 4 also cannot be answered unless students look up the total area of Rondônia. The circle graph gives them the proportion of land use relative to the whole but not an actual measurement. Question 5 can be answered but shows another weakness of circle graphs: unless the circle is very large, it is difficult to show tiny portions.

Activity 3: Bar Graphs

Use Student Handout 1-19

By reading bar graphs students can see comparisons. Bar graphs can give snapshots of change by showing data for an item in various time periods. They can also show a comparison of several items at one time. The bar graphs on Handout 1-19 compare different groups of American Indians on three criteria.

Students will be able to answer questions 1, 3, 4, 5, and 9 quite easily, as bar graphs distill information into easily read bits. By comparing two or all three of the bar graphs, they will be able to answer questions 6 and 10.

Students will not be able to answer question 2, as this bar graph shows a comparison of ten groups of American Indians, all at 1980. While this bar graph does not show change over time, bar graphs can do so. For example, a different bar graph could be drawn showing the Comanche graduation rate in 1960, 1970, 1980, 1990, and so on.

Bar graphs do not show relation to the whole. Students will not be able to answer question 8 based on the graph, because they know only the *percentage* of high school graduation. They do not know how many Comanche and Kiowa people there are. They can calculate the answers to question 7 because Apache and Sioux populations are given in the circle graph. Without the circle graph they would not be able to make these estimations. Help students understand that using different types of graphs can yield more information, as it does here.

Activity 4: Picture Graphs

Use Student Handout 1-20

Picture graphs can show an enormous amount of information. On the picture graph in Handout 1-20, the size of each country shows population relative to other countries, as it would in a bar graph. Comparisons between countries and continents are easy to make. Also each country's population can be seen relative to the world, as sections on a circle graph would show them. Shading shows growth, which is often done using a line graph. A great deal of information can be shown on this graph because the graph makers used shading, composition, and size to depict information.

Students may confuse picture graphs with other types of illustrated graphs, such as bar graphs. There are important differences. Color and size in an illustrated bar graph may make it pleasant to look at, but they add no information beyond what plain bars would have shown. A picture graph uses visual elements to add significant layers of information. Visual elements can be color, shape, shading, size, location, composition, etc.

(You may need to emphasize that this is a graph, not a map. A map attempts to show precise location and geographic size and would likely include rivers, small countries, mountains, and many other features. This graph focuses on population, the general location of that population, and the growth trends of the population.)

Students will be able to answer questions 2, 6, 7, and 8 easily, as this picture graph gives visible comparisons.

With metric rulers students will be able to determine the approximate populations called for in question 4. Remind them that the area of a rectangle equals its length multiplied by its height. Students can break areas into rectangles in order to compute the populations. You may want to have them make a chart of a few equations to help them convert measurements to population. One centimeter square equals how many people? Two millimeters square equal how many people? An overhead transparency of a metric grid that is marked every five millimeters and placed on top of the graph could also be used.

Students will need to see a world map to respond to questions 5 and 10. There are more than forty countries not included in this graph. Like many graphs, this one shows

large amounts easily and small amounts with difficulty, if at all. In responding to question 10, encourage students to compare the visual representation of each country listed. They might write, "Canada must not be very crowded," or "India has a lot more people for its size than most other countries." (Note that this graph was created shortly before the Soviet Union split into several independent countries.)

Question 9 asks students if they can determine the population densities of Canada and Hong Kong. The graph shows the population of each country, not the area or population density. With the graph alone students will not be able to answer question 9. By comparing the graph with a map or chart that shows accurate area, they will be able to get an impression of population density. (The map would have to be large enough to show Hong Kong to respond to this question.)

For question 11, encourage students to realize that people choose to make one type of graph or another based on their purposes. If they want to show changes in growth rate for a particular country, they could create a line graph. If they want to show the order of countries by population, say high to low, they could do so on a bar graph. If they want to show the proportion of the world's population in each country they might design a circle graph.

Activity 5: Graphing Population Density

Use Student Handout 1-20

In this activity students create one set of data, conduct research to find another set of data, combine both sets, and create a new graph. The information they put together is best graphed on a bar or a picture graph. You may want to discuss briefly why circle and line graphs are not appropriate means to show information in this case. (Students are not looking for relation to a whole or change.)

1. Focus on the countries from question 10 in the picture graph activity: Argentina, Australia, Canada, China, El Salvador, Hong Kong, India, Japan, Netherlands, and the U.S.S.R. First have students calculate the populations based on the graph, as they did in question 3.

2. In an encyclopedia or a world almanac, have students find out the area of each country.

3. They can then compute each country's population density (population per unit of area).

4. Ask students to make a bar graph or a picture graph showing these densities.

Activity 6: Comparing Graphs

Use Student Handouts 1-17, 1-18, 1-19

The following exercises allow students to experiment with transferring (or trying to transfer) information from one type of graph to another. The purpose of the exercises is to have students understand more about the appropriate application of the various graphing forms.

Exercise 1

Have students transfer the information from the line graph on oil imports to the United States (Handout 1-17) to a bar graph, with the same years marked. Help them understand that while they see snapshots of the trends in oil imports, they miss the fluctuations between marked years, and they miss unmarked years altogether, in this case 1990.

Exercise 2

Have students transfer the information from the circle graph about land use in Rondônia, Brazil (Handout 1-18) to both a line graph and a bar graph. The line graph might have three lines, with cropland and pasture increasing and forest decreasing. Or they could draw it with two lines, forest and nonforest. Either would have only two time periods. Making a bar graph would involve similar choices.

Discussion

Following each exercise, ask students which of the graph forms represents the data most clearly. There is no correct answer to this. Students will see that graph designers choose how to express their data based on what they want to highlight.

Also, ask students if it would be possible to transfer the information on American Indians (Handout 1-19) to other graph forms. In this case the bar graphs distill the information so much that they fit only that form. Ask students what information would be needed to create a line graph. (Data on these same groups of American Indian in other years.) What information would be needed to create a circle graph? (Population data on *all* groups, or at least on the ones that appear on each bar graph, and the total of all American Indian peoples.)

Activity 7: Scaling Line and Bar Graphs

Use Student Handouts 1-17, 1-19

This short activity helps students see that displaying data in different ways can give different impressions and that graphs can be designed to influence people. It is important that they learn to read scales on bar and line graphs with a critical eye. It is

easy to get an impression of a large change (on a line graph) or a significant difference (on a bar graph) even though the scale range can be meaningless.

Have students reconsider the graph "U.S. Dependence on Foreign Oil" (Handout 1-17). Have them draw new versions of the graph, one with the left axis ranging 0%–100%, and another with the left axis ranging 20%–50%.

- Ask students what they notice. Do the graphs seem to give the same message as the original? If not, what message does each now give?

Have students do the same exercise using the American Indian median family income graph (Handout 1-19). They can make the scales on the left side range $0–$22,000 and $10,000–$100,000.

If you want students to experiment further with this, have them repeat the exercise using graphs they find in a newspaper or in advertisements.

Activity 8: Choosing and Applying Averages and Graphs

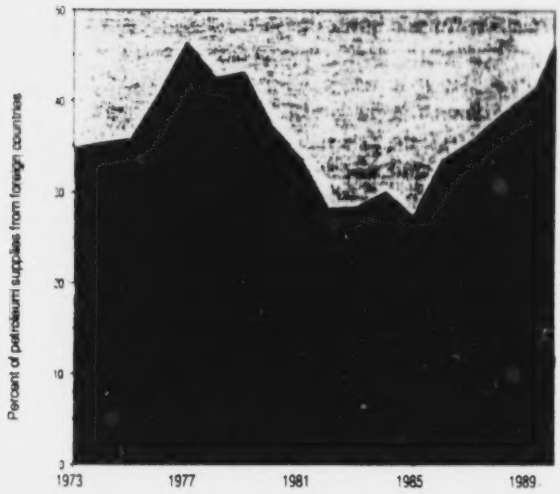
The next task for students is to return to the poll they have been conducting and the data they have been tabulating. Have them apply the techniques they learned about averaging and frequency distributions to describe their data. Ask students to summarize in writing the patterns they notice in their data.

Then have them create graphs that display what they wish to express. Encourage students to try various forms. You may want to require that students use several types of graphs in their projects. These statements and graphs will be part of their final project.

Student Handout 1-17

Line Graphs: U.S. Dependence on Foreign Oil

Read the line graph, "U.S. Dependence on Foreign Oil." Make sure you understand what the numbers on each axis mean.



1. Describe how use of imported oil in the United States has varied since 1973.

2. Approximately, what was the percentage of oil that the United States imported in 1981 and 1989?

Permission granted to reprint graph from "Cool Energy: The Renewable Solution to Global Warming" (Cambridge, Mass.: Union of Concerned Scientists, 1991). Source: The U.S. Department of Energy.

3. What is the most recent year shown in this graph?

4. How much of the country's total energy use involves imported oil?

5. In the time period included in the graph, what were the highest and lowest percentages of oil the United States imported, and when did they occur?

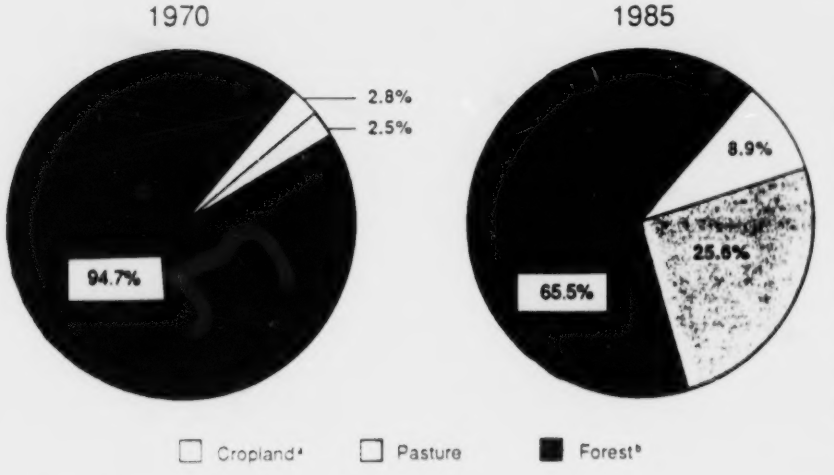
6. In which four-year period did the importing of oil increase the most? (Note that you can select periods that start on years that are marked, or you can determine other four-year periods.) Explain how you determined your answer.

7. Was oil the most used energy source in the United States between 1973 and 1990?

Student Handout 1-18

Circle Graphs: Land Use in Brazil

Study the circle graph "Agricultural Land Use in Rondônia, Brazil: 1970–1985."



a—includes fallow land
b—includes natural pastures

Agricultural Land Use in Rondônia, Brazil: 1970-1985

As world population grows and the demand for food and other consumer products rises, the amount of land converted for human use increases. More land in developing countries is being converted to cropland and pasture each year. Rondônia, a state in the Brazilian Amazon, is an example of an area where millions of acres of virgin forest are being cleared to provide cropland and grazing land.

1. Write five sentences based on the information in these graphs. Describe the land use in Rondônia, Brazil, in 1970 and the change between then and 1985.

2. By what percentage did the forest land decrease over the period of time in the graphs?

Permission granted to reprint graph from "World Environment Data Sheet" (Washington, D.C.: Population Reference Bureau, 1991). Source: The World Bank.

3. When in the fifteen-year span covered in the graphs did the change actually occur?

4. How many square miles of land was pasture in 1970 and in 1985?

5. Assuming that 0.2% of the land is used in a certain way, how would it show up on this graph?

6. Which of the following ratios most closely reflects the proportion of pasture land to total land in 1985?

   1/3 1/2 1/4

7. Estimate a simple ratio for the proportion of cropland in 1985.

8. If you were traveling across Rondônia in 1970, what do you imagine you would see? What would you see in 1985?

9. Why did the graph's makers include two different years?

Student Handout 1-19

Bar Graphs: Census Data on American Indians

Look at the bar graphs on the chart "A Portrait of American Indians."

1. Which group of American Indians had the highest high school graduation rate?

2. Has the high school graduation rate gone down over the last decade? Explain your answer.

3. How much higher is the median family income for the Tlingit than for the Chickasaw and the Kiowa peoples?

4. Compare the median incomes in the "Median family income" graph to the U.S. median average income.

5. Approximately what percent of the Navajo people live in poverty? What percent of the U.S. population lives in poverty?

6. Is the information in the "Median Family Income" graph consistent with the information in the "Poverty Rate" graph?

7. Estimate how many Apache people live in poverty. Estimate how many Sioux people live in poverty.

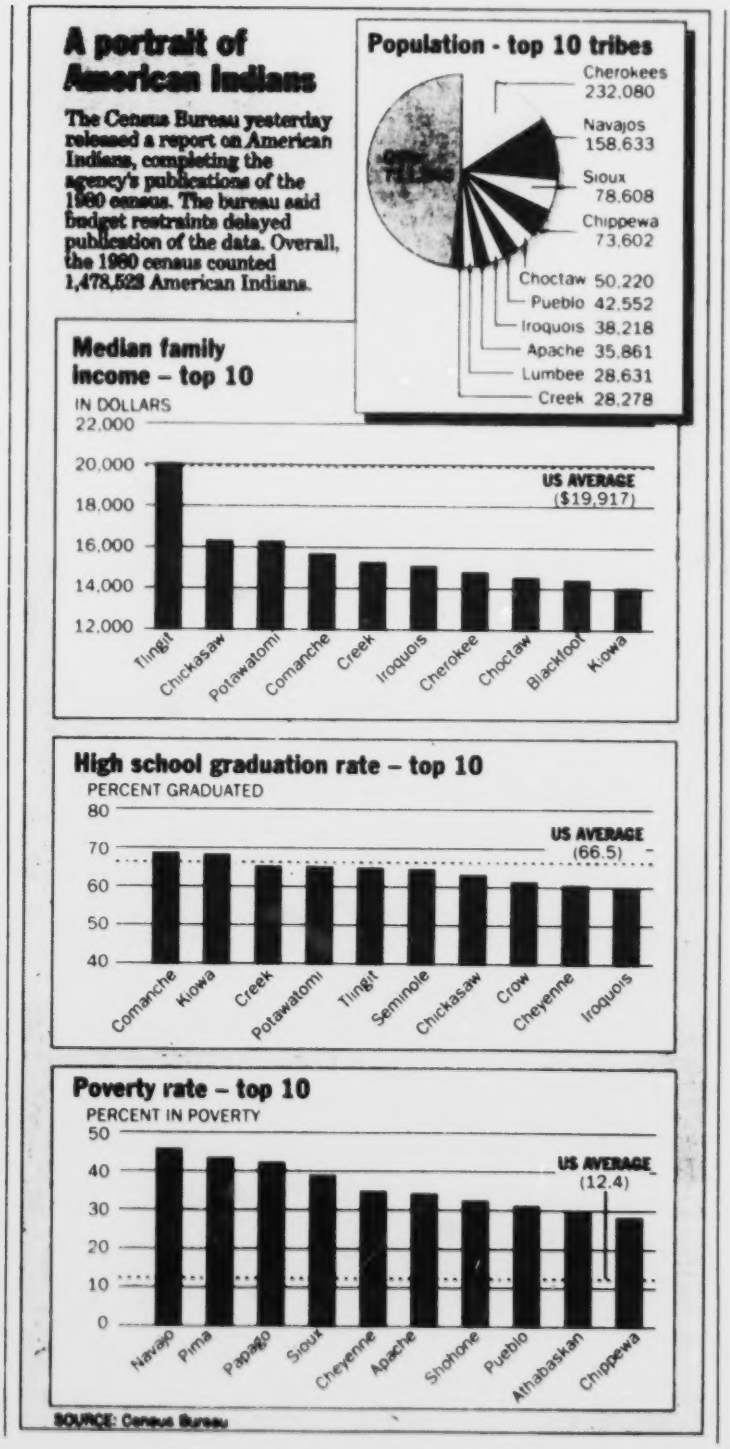
8. How many Comanches have graduated high school? How many Kiowas have done so?

9. Why did the graph's makers include the circle graph?

10. The ten groups with the highest high school graduation rates are shown. How do they compare with the U.S. average?

11. How many of the groups with the highest high school graduation rate are in the top ten of median family income? How many of them are among those groups with the highest poverty rates?

A Portrait of American Indians



Graph from "A Portrait of American Indians," *The Boston Globe*, February 7, 1990. Source: Census Bureau.

Student Handout 1-20

Picture Graphs: World Population

Look at "The Population of States" graph.

1. What does this graph show?

2. At first glance which two countries have the highest population?

3. Note the key showing how much space on the graph equals 1% of the world's population and how much space equals 0.1%. Use the figure for total population given on the graph (5,321 million people) to determine how many people are represented by each of those percentages.

4. Estimate the populations of:

   Egypt
   Nigeria
   Belgium
   Australia
   United States
   Chile
   China

5. Compare this graph with a world map. Name at least three countries that are not included in the graph. Why do you think they were not included?

6. Which continent has the highest population?

7. Which countries have the fastest-growing population?

8. Which countries have populations that are not growing?

9. Is Hong Kong crowded? Is Canada crowded?

10. Compare the size of the following countries on a world map and on the graph. Write a sentence about each country's population.

    Japan

    Kenya

    Australia

    U.S.S.R.

    The Netherlands

    Canada

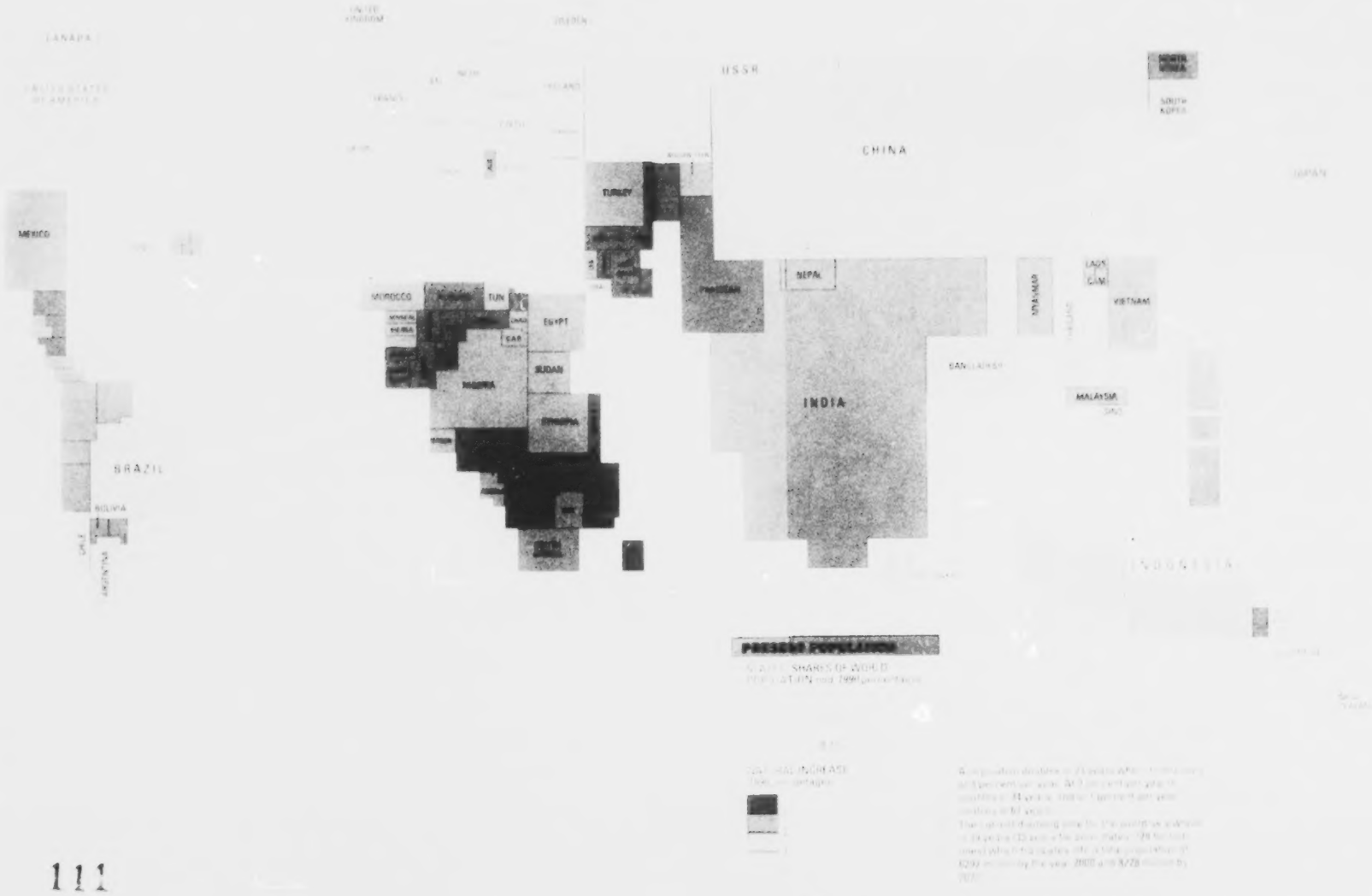
    India

    Argentina

11. Why do you think the graph's designers chose this type of graph to display their information? What is the overall point of the graph? Would other types of graphs have worked to demonstrate the same point?

The Population of States

There are 5321 million people on earth and many more on the way



Permission granted to reprint graph from Michael Kidron and Ronald Segal, *The New State of the World Atlas,* 4th edition (New York: Simon & Schuster, 1991).

BEST COPY AVAILABLE

Interpreting Polls

Overview

A crucial part of the polling process is interpreting the data collected, that is, the process of giving the data value. Interpreting involves both assessing the polling process and drawing conclusions.

An important point to get across to students is that when they interpret data, they should start with *their own responses* to the data. This helps makes interpreting data less overwhelming and helps them see that their own responses count. After all, there is no one "correct" interpretation for the results of a poll. There are usually many valid perspectives.

Interpretation is a way of attaching meaning to statistical or mathematical data within a context. There are many complicating factors that can affect the results or point of view of final statements. If a poll is presented accurately, it will include a detailed and complete presentation of the data along with background information.

Also keep in mind that interpreting data takes time. This point should be emphasized to your students. When they are reviewing information, they should determine how many statements they can make from the data. If it seems that very little can be said about the data, then the information should be reviewed again. Have the students determine if there is enough information to ask questions and whether the information leads to other questions or conclusions. A more in-depth analysis and a greater understanding of the information result from looking at the data in as many ways as possible.

To become more confident and competent in making interpretations, students need the opportunity to practice interpreting data from many different polls. They should be encouraged to find as many possible interpretations for the same data as they can. They should come to realize that data can have different meanings to different people depending upon their point of view. And although data can tell us things, often what the data does not tell us can be just as important.

This section is divided into four parts. Activity 1 focuses on interpreting a single statistic and its context. Activity 2 examines the poll itself and whether it offers background information for evaluation. Activity 3 examines opposing views of the same set of statistics. Activity 4 is the final stage in the students' polling projects.

Vocabulary

review of terms from the entire chapter

Materials

Handouts 1-21 through 1-24, calculators, Handout 1-12 (the checklist already given to students)

Time Needed

four–six class periods

Activity 1: Introducing Interpretation

Use Student Handout 1-21

Knowing the context of numerical information offers us more of a basis for developing initial feelings and impressions and for evaluating and making decisions about an issue or situation. So that your students can understand this better, begin by having them read Handout 1-21. Here they are asked to give their impressions of each item on a list of single statistics. Referring to the data from the handout, pose the questions below. Note that it is useful to ask these questions when beginning to interpret a poll, or any other form of numerical information. You may want to write them on the chalkboard, create an overhead, or make a handout.

- What are your initial reactions to the data? What are your conclusions?
- How do your responses compare to the responses of others?
- What questions do these responses raise about the data?
- Can you imagine different scenarios or situations that could explain the same data equally well?
- Do these scenarios help you to better understand the results? How?
- Does the data prove anything to you?
- What additional information would be important to convey if you were explaining this data to someone?

Through this handout, students have the opportunity to attach meaning to a numerical value, while also considering the context in which the percentage occurs. However, some of the numbers are not complete. For example, the unemployment figures for Massachusetts cause concern, even though we are unable to calculate the number of people represented by the percentage. This is due to the lack of information about how many people are employed.

After the students have read the handout, divide them into groups of two or three and have them discuss the meaning of these percentages and what impression they had of the statistic and the context in which the statistic occurs. For example, they might calculate the number of literate people in China or Afghanistan. They will need to refer to Handout 1-20 to complete this example. Then they might present their findings and impressions of the data to the rest of the class.

Activity 2: Assessing the Process: "Portrait of a Generation"

Use Student Handout 1-22

Another area of interpretation concerns the presentation of the results of a poll. It is important to assess the details of a poll, particularly background information. The following questions can help evaluate the background information presented in a poll.

Questions on Assessing Polls

- Are the results of the poll backed up by statistics and explanations, or are they simply announced?
- How complete are the statistics? How were the statistics gathered?
- What is the poll about?
- Who was polled? Does the article say? What percentage of the population actually responded?
- Who is the author of the poll? What is the author trying to discover?
- What kinds of responses were available to the people questioned?
- Are the results presented in a biased way?
- Should you believe the results?
- What action should you take now that you have the results of the poll?

You may want to write these questions on the chalkboard, make an overhead, or make a handout for your students. In this way, students can refer back to the questions as they examine different polls and try to interpret them.

In Handout 1-22, have your students complete the questions as a way of examining a poll. For students to calculate the percentages they should use the figure 1,521 given in the introductory text for the total number of people interviewed.

In question 4, the numbers for the "Total" column cannot be determined from the information given in the first three columns. To compute these numbers students need to know the number of people in each age group or the proportion of people in each age group. The total figure is a *weighted* average of the individual percentages; for example, 75% is a weighted average of 84%, 82%, and 66% (the "mostly good" percentages for "changes in the role of women"). You could make this clear to students by showing them an example. If the three age groups contained, say, 300, 758, and 463 people, respectively, then the total percentage would be

$$(500 * 84\% + 758 * 82\% + 263 * 66\%) / 1521 = 1216 / 1521 = 80\%.$$

However, if the three age groups contained 50, 758, and 713 people, respectively, then the total percentage would be

$$(50 * 84\% + 758 * 82\% + 713 * 66\%) / 1521 = 1135 / 1521 = 75\%.$$

Note the figure 758 in the footnote, which is the total number of people in the 30–40 category.

For question 5, note also that the number of people in each category *can* be determined from the 75% figure. Just let x represent the number of people in the 18–29 category; then 763 - x represents the number of people in the 41 and older category, so that x can be determined from the equation

$$[x * 84 + 758 * 82 + (763 - x) * 66] / 1521 = 75.$$

Solving this equation gives x = 71 (after rounding). With this figure, the raw numbers can in fact be approximately calculated. Thus, roughly 60, 622, and 457 people answered "mostly good" in each of the three categories, while roughly 8, 99, and 138 people answered "mostly bad." This gives the total percentage 245/1521 = 16% for the "mostly bad" total.

In question 6, students can observe trends among different age groups. Older women grew up in an earlier era, when their roles in society were defined differently.

For question 8, there were many events during the 1960s and 1970s. This question is an interesting way for students to talk to their parents about the history of the times. Also in question 8 the figure 40% represents 403 people (rounded off), and the figure 55% represents 554 people (rounded off).

In question 11, the survey was conducted from April 15 to April 17. April 15 is the day taxes are due, which might have influenced the responses to the questions about the government.

Activity 3: Opposing Points of View

Use Student Handout 1-23

There are times when people are confronted with information that is not clear-cut. Sometimes the conclusions from data can be interpreted in several ways or can lead to opposite points of view. In such situations, students need to review the data and decide which statements may best represent their own opinion. Encourage your students to review the data for accuracy, follow the conclusions closely to see if they match the information being given, look for the source of the data, and then decide which conclusion makes the most sense to them.

In the handout "Justice Department Contradicts FBI on Crime Rate," students will read about two governmental agencies that do not agree on the interpretation of the same figures. It is important for students to follow, carefully, the reasons each agency gives for its conclusions. The questions are designed to support this type of examination. Encourage your students to answer each question thoughtfully and with as full an explanation as possible.

For question 1 in Handout 1-23, students may want to measure crime in several ways. They may suggest using a technique similar to the FBI and thus count crimes reported to several law-enforcement agencies. They may suggest taking a sample of a few urban, suburban, and rural areas, and then predicting what these numbers would look like for the country. Or students may have other ideas for how they might predict the country's crime rate. In any case, allow your students time to explore the opposing views in this article.

In question 4, crime rate measures the overall rate at which crimes occur during the course of a year.

For question 8, students are likely to have differing points of view. Have them support their opinions with information from the article.

In question 9, the responses may vary. One answer is: An increase in the rate could just reflect an increase in the number of crimes being reported. Also, if more agencies are included in the poll, then there is the possibility of the rate increasing.

Activity 4: Margin of Error

Use Student Handout 1-24

Handout 1-24 introduces students to margin of error, a statistical tool that is often reported but not well understood by the general public. "Margin of error" refers to the interval about the true population mean in which a sample mean is expected to be found.

For example, it might be reported from a poll of 1,000 people that 47% of those polled were opposed to gun control, with a margin of error equal to 4%. What is often not reported, but assumed, is the phrase "with 95% confidence." Taken together, these statements mean: think of all possible samples of size 1,000 and imagine asking the same questions and tabulating the analogous results for each of these samples; then 95% percent of the sample results will lie within ±4% of the true population statistic. So if the poll sample happens to be one of the 95% percent of samples in that range of the population mean, then the population mean will be between 43% and 51%. However, it is possible that the pollsters chose a sample that is one of the 5% of samples that do not fall within ±4% of the population mean. At least 5% of the time the results of a poll will not tell us anything about the actual opinions of the whole population.

To understand these concepts, students work with a specific population of six people, and take samples of size three. The population is this small so that students can find all possible samples of size three (there are twenty of them) and can see the relationship of the sample means to the population mean. By plotting the distribution of sample means, students can find confidence intervals. We use the term "confidence interval" to mean the interval about the population mean that includes a given percentage of the sample means of the size being considered. Students are asked to find the margin of error corresponding to 80% and 90% confidence intervals about the population mean.

We emphasize that this activity is intended only to give students the idea of margin of error. They should know what the margin of error means when they see it

reported with a poll result, particularly because it makes clear that poll results have limited validity.

However, students will not learn how to calculate margins of error for their own poll results. The calculations in this activity are manageable and meaningful only when the population mean is known and all possible samples of a given size can be determined. In real situations the population mean is unknown; the whole purpose of taking a sample is to estimate the actual population mean. In this situation the margin of error can be determined only by using theoretical methods from statistics. (Cf. David S. Moore, *Statistics, Concepts and Controversies:* W.H. Freeman, 1991.)

The answers to the questions on Handout 1-24 are given below.

1. The population mean is 15.3.

2,3,4. All twenty samples of size three from ABCDEF, together with the sample means asked for in question 4 are:

ABC-16.3
ABD-15.3
ABE-11.7
ABF-14.3
ACD-18.7
ACE-15
ACF-17.7
ADE-14
ADF-16.7
AEF-13
BCD-17.7
BCE-14
BCF-16.7
BDE-13
BDF-15.7
BEF-12
CDE-13
CDF-19
CEF-15.3
DEF-14.3

7,8. The 80% confidence interval is 13–17.7. The two highest and two lowest sample means do not lie in this range. The corresponding margin of error is ±2.4. This is the larger of the two distances from 15.3 to the endpoints in the 80% confidence interval.

9. The 90% confidence interval is 12–18.7, with margin of error ±3.4 about the population mean.

10. To determine what the statement "62% of Americans support a presidential action, with a margin of error of 3%" means, you first need to know that the 3% margin of error figure refers to the 95% confidence interval. You also need to know what the sample size was for the poll. If the sample size was 1,500, then the statement means:

    of all possible samples of size 1,500, 95% of their means will lie within ±3% of the true population mean.

    Since the population mean is never known in actual practice, this just tells you that with 95% confidence, the population mean lies between 59% and 65%.

Activity 5: Final Project

Students have developed questions, picked samples, conducted their polls, determined averages, and displayed their data. Now they should write a final report that includes intrepretations of their results and a careful explanation of the steps they went through.

They should include the following information in their reports:

- all the data they collected
- frequency distributions
- averages
- graphs
- interpretations (some context in which these responses might have occurred and why)
- assessment of the process by which they constructed and conducted their poll

They can use the checklist (Handout 1-12) as a guideline to help them complete their projects.

Have students present their final projects to their classmates. Make available posterboard, overhead transparencies, or other supplies that will allow them to make very thorough and polished presentations. Students may even want to write articles about their poll results for their school or community newspapers.

Student Handout 1-21

Meaning in Percentages

When we give meaning to a mathematical concept, we use the concept as a way of measuring something, and then we have to decide whether the number is important or not.

In this assignment, react to the numbers and say what impression they make on you. Think carefully about all the possible meanings the percentages might have.

For example, 51% is considered a very low test grade. However, in an election 51% is considered a victory, because it is a simple majority of the votes.

- In northeastern Brazil 22% of the rural children show long-term malnutrition.
- What would your reaction be if 22% of the people in the United States were showing long-term malnutrition? How many people would that be, and what impression would this have on you?
- In the Philippines the richest 20% of the people get 54% of the income; the poorest 20% get 5% of the income.
- The best fossil-fuel generating plants in use have an efficiency rate of about 40%.
- In the spring of 1991 the unemployment rate in Massachusetts was about 9.3%.
- The literacy rate in the country of Afghanistan is 12%, while the literacy rate in China is 70%.

Student Handout 1-22

"Portrait of a Generation"

Read the article below and pay careful attention to the construction of the poll and the data being presented. It may be helpful to recall the earlier questions about assessing a poll.

1. When percentages are given, is the sample size or raw data offered? How many people responded?

2. Are the pollster's questions, or a sense of the pollster's questions, given in the study?

3. How was the poll conducted? (by phone, interview, etc.)

4. How were the numbers for the "Total" sections calculated?

5. Can you calculate the real numbers (raw data) for the section "Changes in the role of women"? Why or why not?

6. The section entitled "Changes in the role of women" contains some interesting data. What do you notice? Offer several explanations for the relationship between the different percentages in the responses and the age group that responded.

7. Who are the Baby Boomers?

8. In the section on government and business, calculate the actual number of responses (raw data) and put them on a bar graph. This study reflects the 1960s and early 1970s. Were there any events at that time that might contribute to these responses? How old were the Baby Boomers at that time? Interview one or more Baby Boomers, and research the 1960s and early 1970s.

9. In the "Better or worse off than their parents?" section, which age group generally feels better off, and which age group generally feels worse off? Why do you think these age groups responded the way they did? What factors could contribute to these responses? Does age have anything to do with the responses? Why or why not?

10. Do you feel better or worse off than your parents? Why? What, if any, trend do you see in these responses?

11. Could the dates during which the survey was conducted have any influence on the outcomes?

Portrait of a Generation

How do Baby Boomers perceive themselves and their times? How do their perceptions differ from those of earlier generations? To find out, TIME *commissioned a poll by Yankelovich, Clancy, Shulman of 1,007 Americans 18 years and older, plus an additional sample of 514 Americans between the ages of 30 and 40.* Some of the more significant results:*

Were the '60s and '70s good or bad?

All things considered, 61% of those ages 30 to 40 said the changes that occurred during the 1960s and 1970s were more constructive for people their age, while 31% said the changes were more destructive. How they and other generations rated two of the most revolutionary changes:

	18-29	30-40	41 & older	Total
Changes in the role of women:				
Mostly good	84%	82%	66%	**75%**
Mostly bad	11%	13%	20%	**16%**
Changes in attitude toward sexual conduct:				
Mostly good	50%	41%	25%	**36%**
Mostly bad	44%	51%	60%	**53%**

More conservative, but still fighting the Establishment

Sixty-four percent of the Baby Boomers ages 30 to 40 said they have become more conservative in their political views, while 27% said they have become less conservative. Although 34% smoked marijuana "regularly" or "sometimes" in the past, only 11% do now. But if anything, it appears that the Baby Boomers have become less trusting of authority and big institutions since the 1960s and early '70s.

More trusting of Government:	**40%**
Less trusting of Government:	**55%**
More confidence in Big Business:	**33%**
Less confidence in Big Business:	**64%**

*The survey was conducted by telephone April 15 to 17. The potential sampling error is plus or minus 3% for the entire population, and plus or minus 4% for the 758 individuals between 30 and 40 years of age who were interviewed.

Assuming new responsibilities

Eighty-nine percent of 30- to 40-year-old Boomers feel they have become "more responsible members of society." Forty-three percent believe they have compromised their true beliefs. Most ascribed their changes to homelife: 74% said that "assuming new family responsibilities" was a very important factor in changing their beliefs and values.

Better or worse off than their parents?

Of the 30- to 40-year-olds, 63% said that in general they are better off economically than their parents, while only 28% said they are worse off. Questioned on specifics, they painted a more negative picture:

	18-29	30-40	41 & older	Total
The cost of housing:				
Better off	27%	22%	41%	**33%**
Worse off	66%	70%	45%	**56%**
The cost of living:				
Better off	39%	38%	50%	**44%**
Worse off	53%	53%	38%	**46%**
The ability to get by without two wage earners in the family:				
Better off	35%	29%	40%	**36%**
Worse off	53%	61%	41%	**49%**

Most important, 68% of Baby Boomers said they place more emphasis than did their parents on doing interesting things in their lives. Asked whether they have more freedom to do what they want, they were overwhelmingly positive:

	18-29	30-40	41 & older	Total
Better off	81%	77%	68%	**74%**
Worse off	10%	11%	13%	**12%**

Permission granted to reprint "Portrait of a Generation" from *Time Magazine,* May 19, 1986, p. 35.

Student Handout 1-23

"Justice Department Contradicts FBI on Crime Rate"

The article below contains some interesting information on the interpretation of statistics. Read the article carefully, and consider how each governmental agency came up with its conclusions.

1. How would you measure crime for one year?

2. What did the Justice Department conclude about the 1985 crime rate?

3. What did the FBI's annual crime report say about the crime rate?

4. What does the crime rate measure?

5. Compare the methods that the FBI and the Bureau of Justice Statistics use to find the crime rate.

6. Who found a higher crime rate? Is there a significant difference between the two rates?

7. The FBI crime report has been criticized. Why?

8. Which method (the FBI's or the Justice Department's) do you think represents actual crime more realistically? Why?

9. The FBI reports a 5% increase in the crime rate during 1985. Is an actual increase in crime the only factor that could explain this rate increase? If not, what other factors could contribute to the rate increase?

10. List three questions directed at both groups of researchers that would help you better understand their studies.

Justice Department Contradicts FBI on Crime Rate

by Jeff Nesbit

WASHINGTON—Crime in the United States last year fell to its lowest level in 13 years, the Justice Department reported yesterday.

An estimated 34.9 million crimes were committed in 1985, down from more than 35.5 million in 1984 and a peak of 41.4 million in 1981, the Bureau of Justice Statistics said in its annual Uniform Crime Survey.

Those findings contrasted sharply with the FBI's annual Uniform Crime Report issued in late July, which showed an increase of nearly 5 percent in crimes reported during 1985.

The two reports attempt to measure the same thing—whether crime is on the rise or falling off from year to year—but take entirely different approaches.

The widely circulated FBI report is based on crime reports provided by nearly 16,000 local law enforcement agencies.

The Bureau of Justice Statistics interviews more than 100,000 people twice a year to see how their lives have been touched by crime during the course of the year and, from the results, projects the number of crimes committed in the country.

The FBI's 1985 crime report said that about 12.5 million crimes were known to police that year, compared to the nearly 34.9 million crimes the Bureau of Justice Statistics said were actually committed last year.

The FBI doesn't count crime. Two-thirds of the crimes in this country are never reported, said a Bureau of Justice Statistics official who asked to remain anonymous. "The FBI gets their numbers from 16,000 law enforcement agencies. How can you trust those figures?"

A preliminary FBI crime report that will be issued later this month "will show that crime has gone up 10 to 15 percent" during the first six months of 1986, a crime index trend that is not reflected in the Bureau of Justice Statistics own report, this official said.

"The two reports are very different" he said. "We estimate the total number of crimes committed. . . . Ours is a better report."

The FBI crime report has come under attack in the past because it doesn't gauge unreported crimes and, critics say, law enforcement agencies sometimes under- or over-report crimes in their region for political reasons.

The FBI has tentatively begun to make some major changes in the program but it will be "years and years" before they are reflected in the report, said Harper Wilson, an FBI spokesman for the uniform crime reporting division.

The latest Bureau of Justice Statistics' National Crime Survey, which has been conducted every year since 1973, found that since 1981 the number of violent crimes has dropped 12 percent, personal thefts have fallen 15 percent and household crimes (burglary and auto theft) have declined 18 percent. In the last year alone, the study showed, violent crime was down 3.3 percent, personal thefts 2.3 percent and household crimes 1.1. percent.

In Boston, police recorded a total of 68,998 crimes in 1985 up 7.3 percent over the previous year.

The greatest jump came in the number of manslaughters, which doubled, from 8 to 16. Other violent crime also saw significant increases: rape rose 15 percent to 538, aggravated assault 13 percent to 5,076, and murder over 7 percent to 88. Robbery increased nearly 13 percent to 6,278; larceny nearly 11 percent to 27,026; motor vehicle theft nearly 4 percent to 18,474; and burglary by less than 1 percent to 11,539.

Reprinted by permission of Knight-Ridder Tribune News, Washington, D.C. Appeared in *The Boston Globe*, October 9, 1986.

Student Handout 1-24

Margin of Error in Sampling Television Watching Time

Six students, named A, B, C, D, E, F, were asked how many hours a week they spend watching television. The results are given below.

Hours/week spent watching television

A	B	C	D	E	F
15	12	22	19	8	16

Treat this group of six students as your population, and experiment with sampling from the population.

1. First find the mean number of hours of television watched per week in this population of six students. This is the population mean.

2. Now take a sample of size three, say A, B, and E. Find the mean number of hours spent watching television by *just these three students*. This mean is called the sample mean. How does the sample mean compare to the population mean?

3. Now make a list of all possible samples of size three from this population. Try doing this systematically, starting with ABC, ABD, ABE, ABF, ACD, etc. How many different samples (combinations of three students) are there?

4. For each of the combinations of 3, find the sample mean.

5. Now make a bar graph of the sample means. The numbers that occur as a sample mean should appear on the horizontal axis, and the frequency of each sample mean should appear on the vertical axis.

6. Locate the population mean on the horizontal axis of your bar graph.

7. Identify the 80% of sample means that are closest to the population mean. Which sample means does this not include? What is the range of this group of 80% of sample means?

   We are calling this range the *80% confidence interval*. It means that 80% of the time you take a sample of size three, you can be confident that the sample mean falls within this range about the population mean.

8. Find the largest distance from the population mean to the sample means in the 80% confidence interval.

   Plus or minus this distance is called the *margin of error*. When you take a sample of size three, then 80% of the time the sample mean will be within the margin of error of the true population mean.

9. Now find the 90% confidence interval and the corresponding margin of error.

10. When a poll is reported in the news or in a magazine, the margin of error usually refers to the 95% confidence interval. Such a poll result might be reported as follows:

    62% of Americans support a presidential action, with a margin of error of ±3%.

    What does this mean? What else do you need to know to give a complete answer to this question?

Part 2 Counting on People

Introduction

In Part 2 most of the mathematics relates to information obtained from the United States census. The wealth of information available in census documents allows for many different mathematical activities on current and historical trends in the population of the United States.

Part 2 begins with an activity in which students take a census of their class, so that they have some feel for what the census is. Next, there is a reading that explains some of the history of the census, as well as some of the current controversy about undercounting. As part of this lesson students are asked to estimate the number of people it takes to complete the census. This activity introduces students to the art of intelligent estimating.

If you are using Part 2 without having taken your students through Part 1, you may want to use the discussions "What Does This Have to Do with Math?" and "Who Needs Math Skills in Social Studies?" from Part 1. These discussions could be inserted just after the introductory activities or elsewhere in Part 2. You will need to change the questions so they relate to the census instead of polling.

In "The Census and Reapportionment" students work with small groups of states to investigate several methods of apportioning among the states the 435 representatives in the U.S. House of Representatives. Apportionments, along with the allocation of tax money, is the main reason the census is taken every ten years. Two methods are explained; a modification of one of these methods is currently used by Congress to do the reapportioning. The other method was proposed in 1981 but not accepted by Congress.

The reapportionment exercises illustrate that there can be many mathematical procedures for a given task. The undercounting controversy also demonstrates that the choice of procedures can be politically motivated. The last activity of the lesson illustrates the effects of undercounting on reapportionment. The mathematics involved in this lesson is simple (rounding, working with decimals), but the methods are subtle.

The lesson on redistricting illustrates the next step of the reapportionment process using a fictional state called Billows. Students are given a map of Billows and required to divide the state into voting districts, based on one of several different scenarios. Again, the mathematics is affected by political choices. The second part of the lesson introduces students to the idea of population density by having them study a map of the voting districts of their own states (available at any federal repository library).

The next set of activities explores the important concept of a rate of change through several class discussions and exercises, which include computing rates of change, graphing data and seeing how the graph reflects positive and negative rates of change, and coming up with a formula for rates of change . This gives students the opportunity to invent formulas and express the mathematics in their own way. In the culminating activity of the lesson, students use census data to study trends since 1940 in years of school completed by people in the United States of different races and both genders.

"Doubling Time" takes the ideas of the last lesson further and introduces the concept of exponential growth. The first activity is based on an article by Donella Meadows

about environmental resource depletion. There is also a physical exercise intended to convey a kinesthetic sense of exponential growth, as well as an activity in which students graph the change of the U.S. population since 1790 and use this to compute doubling times. This activity can be extended by investigating the doubling times of other countries.

In "Cross-Referencing Census Data" students have the opportunity to put together information from different tables and charts. They do this by brainstorming things they can deduce from the chart as well as things they cannot know because some crucial piece of information is missing. This lesson prepares students for the type of thinking involved in problem solving and in historical and social research, where the problem solver or researcher must distinguish between important and extraneous information and understand how the given information fits together. Students can get by in this lesson with knowing how to solve proportions, but it is also an opportunity for introducing linear equations (two linear equations in two variables).

In the final lesson students are asked to use "Journey to Work" data from the census to design a rail transportation system for Los Angeles County. This requires them to use many different mathematical techniques in one context. It is also good practice in logical reasoning and gives the students another opportunity to be creative with mathematics they already know. The key is to give the students all the materials they might need and let them create. Teachers can choose from two options for taking the lesson further: having the students design a system that stays within a budget and/or determining traveling times as part of the design process.

Throughout Part 2 one emphasis is communicating mathematical ideas, through class discussions, small group activities, and student presentations. Students should be challenged to talk and write mathematics often. As with any language, the best way to learn it is to use it in many different ways. High standards should be set for both written and oral work: students should be encouraged to be very clear, to define their terms, and to explain their methods and rationales carefully. This is as much a part of good mathematics as are proper use of specific techniques and formulas.

Other mathematical themes that Part 2 emphasizes are using mathematics as a decision-making tool and formula making. It is important for students to understand that they can make mathematical concepts work for them and can be creative in using mathematical ideas. Thus, some of the activities, such as the Los Angeles transportation project, do not specify what type of mathematical tools should be used. It is up to the students to discover what they need to use, what helps them solve their specific problems. In the formula-making activities students can be encouraged to understand a specific idea by coming up with their *own* formula in their *own* terminology. They will be much better prepared to understand and use the formula if they have found it (or invented it) themselves. Students develop formulas in several activities; among them are activity 5 in "The Census and Reapportionment," Activity 4 in "Rates of Change," and option 2 of the Los Angeles transportation project activity.

Classroom Census

Overview

As an introduction to the U.S. census, students will experiment with taking a census of their class.

Skills

experimenting with data, calculating, graphing, using percentages

Vocabulary

raw data, percentage, census

Materials

paper, computers (optional for graphing), calculators

Time Needed

one class period

Activity: Taking a Class Census

1. Introduce students to the concept of a census: a quantitative description of a group of people.

2. Ask students how they would quantify themselves in raw data and in percentages. They can do these calculations with or without calculators.

   - How many people are in the class?
   - How many are present versus absent? What would those numbers be if expressed as percentages of the class?
   - How many have birthdays in each of the twelve months? What are those numbers in percentage form?
   - How many have brown eyes? black eyes? blue eyes? green eyes?
   - How many are right-handed and how many are left-handed?
   - How would students categorize their footwear (by color, as high top and low top, or as sneaker and nonsneaker)? How would they count their shoes in raw data and in percentages?

   Have them think of several more ways to categorize and count themselves.

3. Have students make bar graphs or line graphs (on paper or on computers) to represent some of these counts. If students graph attendance or another attribute that changes from day to day, have them count it and graph it for a few days. Have them show different days on the same graph so they can see the change.

4. Explain that the rest of the lessons will work with data that has been collected by the U.S. Census Bureau, data which represents all U.S. citizens in hundreds of different ways.

The Census: History and Controversy

Overview

The reading in this chapter offers students information and background on the U.S. census, including the mandate in the U.S. Constitution to conduct a decennial census and controversy about how the counting is done. Following the reading is an extended estimating activity based on the question "How many census employees does it take to gather information from 106,000,000 households?"

Skills

estimating, logical reasoning, using large numbers

Vocabulary

census, decennial, overcounting, undercounting, enumerators, sample, population

Materials

a copy of Handout 2-1 for each student, calculators

Time Needed

two–four class periods

Activity 1: Class Discussion

Use Student Handout 2-1

This activity involves both a reading and a discussion. Have students read the first four pages of Handout 2-1 as a homework assignment. Below are questions for a discussion after students have read the material, some of which require mathematical calculations. Following the handout is an activity and a topic for further research.

Questions for Discussion of Handout 2-1

- How does the statement from the U.S. Constitution imply that a census is required?
- What is currently the average number of people living in a housing unit in the United States? (Students will need to know that the total U.S. population, as counted by the 1990 census, is 248.7 million.)
- How many households is 17% of the households in the U.S.?
- Some people say that politics affects the census count; others say not. What do you think? (Encourage students to think about the earliest *and* most recent history of the census.)

Note: the exercise on reapportionment (later in this part) includes questions about how the undercount mentioned in Handout 2-1 affects representation in the U.S. House of Representatives.

Going Further

Included in Handout 2-1 are two pages that may lead to more discussion and greater understanding about how the census is conducted and what information it gathers.

The first page is "Sample Questions from the Census." Students can fill out the questionnaire and tally their responses. Or they can just make observations about the variety of questions that are asked.

The second page shows the results of the first census of the United States. We include it as a historical reference that might lead to an interesting discussion.

Further historical documents of this kind are available in any federal repository library.

Activity 2: Estimating Enumerators

Divide students into small groups to explore the following question.

Pretend that you have just been appointed the director of the Census Bureau. It is now 1989 and you have to make sure you have enough people on your staff to conduct the decennial census in the year 1990. How many people will you need to gather information from 106,000,000 households?

This is an open-ended question. The goal is not to get any particular answer. Rather, it is to develop reasoning skills. Though there is a literal answer to this question, it is more important that students play with the question than worry about getting a "correct" answer. (See the article by Joan and Marc Ross on Fermi questions listed in the references.)

Students start with the fact that there were 106 million households in the United States in 1990, together with the information they learned from Handout 2-1. The students' task is to build a logical sequence of assumptions and ideas leading them to construct an interesting estimate of the total number of people that were needed to complete the 1990 census.

Pose the question to your students as a game, with the following rules:

1. Students may make use of the information in Handout 2-1.

2. They may make reasonable assumptions as they go in order to have a basis for the steps they take in building their estimate. They must keep careful track of all their assumptions.

   Encourage them to use their imaginations and be fearless in making assumptions, as long as they have clear reasons for making these assumptions.

3. They may ask questions. However, each group may not ask more than a total of *five* questions.

   This rule encourages students to think carefully about the kind of information they need and to negotiate with each other to pick the most powerful questions. Try to avoid situations in which the students ask continual questions without choosing the best questions to ask.

   Real information regarding typical questions asked is written below. Offer students answers to their questions, once they have clearly articulated and agreed upon their questions as a group. For other questions, feel free to offer your own assumption or have students make their own.

   They may come up with different questions from those listed below or similar questions in a different order. Encourage them to repeatedly ask themselves *what they need to know to build toward their estimate.*

4. The students must record all the assumptions, calculations, and questions or statements that form their argument, in such a way that someone who was not part of their group can follow their reasoning.

   Merely guessing is not acceptable; it will not lead to a "reasoned" estimate. When the groups have completed their estimations, have each group explain their estimates and their rationales.

Additional Information

- The census is conducted from April to June in a given year.
- Census forms are mailed out in late March so people have them on Census Day, April 1.
- Questionnaires from approximately 70,000,000 households are returned by mail within two weeks of Census Day, April 1.

Questions that Students Might Consider

Many aspects of completing the census are not covered in these questions. These are just some ideas for getting started.

- How long would it take for a census employee to interview someone and ask him or her the Census questions?
- How many households could one employee visit in one day if he or she works eight hours and everyone is home? Remember to consider travel time between households, which can be in urban or rural areas.
- How many households could an employee visit if he or she works from 4:00 to 8:30 p.m. on weekdays only? How many interviews could he or she fit in if he or she also worked 10:00 a.m. to 5:30 p.m. on Saturdays?
- If census employees find someone home at about 25 percent of the households in a given neighborhood, in any visit to that neighborhood, how many trips to that neighborhood might they have to make?
- If 70,000,000 questionnaires are returned by mail within two weeks of April 1, how many households need to be visited?
- If the Census Bureau gives everyone two weeks following April 1 to mail in their forms, and needs to be done interviewing everyone by the end of June, how many weekdays are there to accomplish the task?
- How many employees might be needed to count homeless people living outside shelters across the United States?
- How many people might be needed to mail 106,000,000 forms?
- How many people might be needed to enter data into computers from 106,000,000 forms? Can some of the data be scanned by machines?

- Where in the United States are there people who do not speak or read English? How many bilingual people might be needed to translate forms or conduct interviews?

Going Further

Have students ask their school principal or superintendent if their school system receives federal money. Find out how much is received, from what particular sources (departments, grants, programs, etc.), and how the amounts are determined. Are any of the amounts based on population? They can also call their mayor's office or town hall with the same questions.

Student Handout 2-1

The U.S. Census: History and Controversy

Did you ever wonder where news reports get their information about the number of people living in the United States? Or how you could find out how many people in the country have a particular ethnic heritage? Where would you go to find the number of people who were employed in the country in 1950 or how many people owned cars in 1970?

All of this information comes from the Census Bureau, which is part of the U.S. Department of Commerce. The Census Bureau conducts a survey of population and housing every decade. This survey is called the U.S. Government Census, or just census for short. The census is a poll, though it is very different from opinion polls. Rather than counting opinions, the census counts people and all sorts of different things that people are or do or have. The census counts how many of us there are, where we are, how we get to work, what kinds of houses we live in, how much formal education we have, and what our ancestry is.

Federal, state, and local governments, businesses, organizations and individuals use this data to make many decisions. Here are some examples of how census data has been used[1]:

The high number of working women with small children in a community, revealed by a study of census data, prompted community leaders in a midwestern city to seek and win approval for a community center.

The Big Brothers of America (Chicago) organization used census data on the characteristics of young males—income, education, occupation, etc.—to estimate the number of potential volunteers in selected metropolitan areas.

In Newport News, Virginia, a church official used age data in planning a new church; a high proportion of elderly meant building a structure of one story instead of two.

During a severe heat wave, public health officials in St. Louis used census facts to locate neighborhoods with large numbers of the elderly; city workers went door to door, convincing many of the elderly to go to a "cooling center."

---

[1] This information is from "Why Census Information Is Vital to Communities," Bureau of the Census D-3224.7, U.S. Department of Commerce, Spring 1989.

In San Francisco transportation planners used census information to select bus routes, subway stops, and highways that needed widening.

The census is required by the U.S. Constitution in Article I, Section 2:

> Representatives and direct taxes shall be apportioned among the several States which may be included in this Union, according to their respective numbers, which shall be determined by adding to the whole number of free persons, including those bound to service for a term of years, and excluding Indians not taxed, three-fifths of all other persons. The actual enumeration shall be made within three years after the first meeting of the Congress of the United States, and within every subsequent term of ten years, in such manner as they [Congress] shall by Law direct.

In these original instructions free people and people temporarily bound to service, such as indentured servants, counted as whole persons; American Indians, who were not taxed, were not counted at all; and other people, such as slaves, counted as three-fifths of a person each. These instructions were superceded by the thirteenth and fourteenth amendments to the Constitution. Ratified in 1865 and 1868, these amendments abolished slavery and granted citizenship to all persons born in or naturalized to the United States.

The census is supposed to include every man, woman, and child in the United States. Census questionnaires are mailed to every household, about 106,000,000 housing units. There are two different questionnaires. The short form is mailed to 83% of the households. It asks seven questions about each person in the household and seven questions about the household: each person's name, relationship to each other, sex, race, marital status, how many rooms their home has, whether they own or rent it, how much the home would sell or rent for, and a few others.

The remaining 17% of the households receive the long form, which asks fifty-nine questions, some with several parts and some that need to be answered separately for each member of the household. In addition to asking all the questions on the short form, the long form asks how much education each person in the household has, what his or her ancestry is, whether and where each person works, how each person gets to work, what time each person leaves home to get there, what each person's income is, whether the household has complete plumbing and kitchen facilities, the number of cars owned by people living in the household, how much they spent last year on electricity, gas, water, oil, and other fuels.

Some of the data the Census Bureau produces is based on trying to count everybody, and some is based on a sample of 17% of the households. In towns or other governing units with 2,500 people or fewer, one out of two households gets the long form. In

towns of more than 2,500 people, one out of six households gets the long form. The households receiving the long form are picked randomly from each town or city.

People who own more than one home, however, and receive and fill out the form for each home, will be counted more than once, or *overcounted*. Those who receive but do not fill out the census form will be visited by a Census Bureau employee (or volunteer), who will help them fill it out. If the Census Bureau cannot find them, they will be *undercounted*.

There are many people vulnerable to undercounting: homeless people, people who aren't able to read, or people who don't read English. These people might not get a form or might not be able to fill it out if they do receive one. There are people who don't fill out the form because they think the information will be available to people or agencies they fear. For example, some poor families have more people living in their home than are supposed to and are afraid that the landlord might find out. Other people are in this country illegally and fear the immigration authorities. The information collected from any one household is confidential—no person or agency other than the Census Bureau can read the forms—but some people don't know that or are afraid that it isn't really true.

Undercounting happens more often than overcounting and is very serious. Based on population, the government determines how to apportion the 435 seats in the U.S. House of Representatives and how to spend an estimated $59,000,000,000 annually.[2] The Census Bureau has been criticized for missing too many people in certain groups. After its 1990 census, the Census Bureau contacted 165,000 households and, based on that sample, determined that approximately 5,300,000 people weren't counted. The Census Bureau director, Dr. Barbara Everitt Bryant, recommended that the census figures be adjusted because of the undercount, but Secretary of Commerce Robert Mosbacher decided against any adjustment. Mr. Mosbacher's undersecretary, Michael Darby, explained: "one of the big reasons the secretary was unable to adjust is we don't know how to do it....We don't know where the people are who we didn't find."[3]

Several cities and organizations are suing the government to force a reconsideration of the adjustment. Many leaders feel that the lack of an adjustment disproportionately hurts minority and poor people, particularly those living in cities. David N. Dinkins, the mayor of New York City, was quoted as saying: "To shortchange the American people in this matter—and in particular our nation's cities, which suffer greatly due to the undercount—is nothing less than statistical grand larceny. The census is the very

---

[2] Felicity Barringer, "Commerce Dept. Declines to Revise '90 Census Count," *New York Times*, July 16, 1991, p. 1, 16A.
[3] Ibid.

cornerstone of our democratic system. If we fail to do our best to ensure all Americans are counted, we are failing to live up to the promise of our democracy."[4]

Mr. Dinkins and Mayor Raymond Flynn of Boston, president of the U.S. Conference of Mayors, said:

> Mr. Mosbacher's decision smacked of partisan politics. They said minorities most likely to be represented by Democrats were most severely undercounted. They also said that poor people from minority groups are the most seriously affected by undercounting. "The message to the American people is the poor don't count," Mr. Flynn said, "so don't count the poor." Mr. Dinkins, New York City's first black mayor said: "There was a time when folks who looked like me were counted as three-fifths of a person. This is not far removed from that."[5]

Had the numbers been adjusted it would have been the first time in the history of the census. The question of how people are counted, however, has been part of the history of the census from its beginning.

---

[4] Ibid.

[5] Felicia R. Lee, "No Census Adjustment Hits Big Cities Hard," *The New York Times*, July 16, 1991, p. A16.

Sample Questions from the Census

PERSON 1	PERSON 2
Last name	Last name
First name — Middle initial	First name — Middle initial

START in this column with the household member (or one of the members) in whose name the home is owned, being bought, or rented.

If there is no such person, start in this column with any adult household member.

If a RELATIVE of Person 1:
- ○ Husband/wife
- ○ Natural-born or adopted son/daughter
- ○ Stepson/stepdaughter
- ○ Brother/sister
- ○ Father/mother
- ○ Grandchild
- ○ Other relative

If NOT RELATED to Person 1:
- ○ Roomer, boarder, or foster child
- ○ Housemate, roommate
- ○ Unmarried partner
- ○ Other nonrelative

- ○ Male ○ Female

- ○ White
- ○ Black or Negro
- ○ Indian (Amer.) (Print the name of the enrolled or principal tribe.)
- ○ Eskimo
- ○ Aleut

Asian or Pacific Islander (API)

- ○ Chinese ○ Japanese
- ○ Filipino ○ Asian Indian
- ○ Hawaiian ○ Samoan
- ○ Korean ○ Guamanian
- ○ Vietnamese ○ Other API
- ○ Other race (Print race)

a. Age — b. Year of birth

a. Age	b. Year of birth
0 ○ 0 ○ 0 ○	1 ● 8 ○ 0 ○ 0 ○
1 ○ 1 ○ 1 ○	9 ○ 1 ○ 1 ○
2 ○ 2 ○	2 ○ 2 ○
3 ○ 3 ○	3 ○ 3 ○
4 ○ 4 ○	4 ○ 4 ○
5 ○ 5 ○	5 ○ 5 ○
6 ○ 6 ○	6 ○ 6 ○
7 ○ 7 ○	7 ○ 7 ○
8 ○ 8 ○	8 ○ 8 ○
9 ○ 9 ○	9 ○ 9 ○

- ○ Now married ○ Separated
- ○ Widowed ○ Never married
- ○ Divorced

- ○ No (not Spanish/Hispanic)
- ○ Yes, Mexican, Mexican-Am., Chicano
- ○ Yes, Puerto Rican
- ○ Yes, Cuban
- ○ Yes, other Spanish/Hispanic (Print one group, for example: Argentinean, Colombian, Dominican, Nicaraguan, Salvadoran, Spaniard, and so on.)

If this person is a female —

20. How many babies has she ever had, not counting stillbirths? Do not count her stepchildren or children she has adopted.

None 1 2 3 4 5 6 7 8 9 10 11 12 or more

○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○

21a. Did this person work at any time LAST WEEK?

- ○ Yes — Fill this circle if this person worked full time or part time. (Count part-time work such as delivering papers, or helping without pay in a family business or farm. Also count active duty in the Armed Forces.)
- ○ No — Fill this circle if this person did not work, or did only own housework, school work, or volunteer work. — *Skip to 25*

b. How many hours did this person work LAST WEEK (at all jobs)? Subtract any time off, add overtime or extra hours worked.

Hours

23a. How did this person usually get to work LAST WEEK? If this person usually used more than one method of transportation during the trip, fill the circle of the one used for most of the distance.

- ○ Car, truck, or van
- ○ Bus or trolley bus
- ○ Streetcar or trolley car
- ○ Subway or elevated
- ○ Railroad
- ○ Ferryboat
- ○ Taxicab
- ○ Motorcycle
- ○ Bicycle
- ○ Walked
- ○ Worked at home — *Skip to 28*
- ○ Other method

If "car, truck, or van" is marked in 23a, go to 23b. Otherwise, skip to 24a.

b. How many people, including this person, usually rode to work in the car, truck, or van LAST WEEK?

- ○ Drove alone
- ○ 2 people
- ○ 3 people
- ○ 4 people
- ○ 5 people
- ○ 6 people
- ○ 7 to 9 people
- ○ 10 or more people

24a. What time did this person usually leave home to go to work LAST WEEK?

a.m. / p.m.

b. How many minutes did it usually take this person to get from home to work LAST WEEK?

Minutes — *Skip to 28*

12. How much school has this person COMPLETED? Fill ONE circle for the highest level COMPLETED or degree RECEIVED. If currently enrolled, mark the level of previous grade attended or highest degree received.

- ○ No school completed
- ○ Nursery school
- ○ Kindergarten
- ○ 1st, 2nd, 3rd, or 4th grade
- ○ 5th, 6th, 7th, or 8th grade
- ○ 9th grade
- ○ 10th grade
- ○ 11th grade
- ○ 12th grade, NO DIPLOMA
- ○ HIGH SCHOOL GRADUATE - high school DIPLOMA or the equivalent (For example: GED)
- ○ Some college but no degree
- ○ Associate degree in college - Occupational program
- ○ Associate degree in college - Academic program
- ○ Bachelor's degree (For example: BA, AB, BS)
- ○ Master's degree (For example: MA, MS, MEng, MEd, MSW, MBA)
- ○ Professional school degree (For example: MD, DDS, DVM, LLB, JD)
- ○ Doctorate degree (For example: PhD, EdD)

13. What is this person's ancestry or ethnic origin? (See instruction guide for further information.)

(For example: German, Italian, Afro-Amer., Croatian, Cape Verdean, Dominican, Ecuadoran, Haitian, Cajun, French Canadian, Jamaican, Korean, Lebanese, Mexican, Nigerian, Irish, Polish, Slovak, Taiwanese, Thai, Ukrainian, etc.)

H2. Which best describes this building? Include all apartments, flats, etc., even if vacant.

- ○ A mobile home or trailer
- ○ A one-family house detached from any other house
- ○ A one-family house attached to one or more houses
- ○ A building with 2 apartments
- ○ A building with 3 or 4 apartments
- ○ A building with 5 to 9 apartments
- ○ A building with 10 to 19 apartments
- ○ A building with 20 to 49 apartments
- ○ A building with 50 or more apartments
- ○ Other

H14. Which FUEL is used MOST for heating this house or apartment?

- ○ Gas: from underground pipes serving the neighborhood
- ○ Gas: bottled, tank, or LP
- ○ Electricity
- ○ Fuel oil, kerosene, etc.
- ○ Coal or coke
- ○ Wood
- ○ Solar energy
- ○ Other fuel
- ○ No fuel used

H15. Do you get water from —

- ○ A public system such as a city water department, or private company?
- ○ An individual drilled well?
- ○ An individual dug well?
- ○ Some other source such as a spring, creek, river, cistern, etc.?

H16. Is this building connected to a public sewer?

- ○ Yes, connected to public sewer
- ○ No, connected to septic tank or cesspool
- ○ No, use other means

First Census of the United States

Population of the United States as Returned at the First Census, by States, 1790

DISTRICT.	Free white males of 16 years and upward, including heads of families.	Free white males under 16 years.	Free white females, including heads of families.	All other free persons.	Slaves.	Total.
Vermont	22,435	22,328	40,505	255	[1] 16	[1] 85,539
New Hampshire	36,086	34,851	70,160	630	158	141,885
Maine	24,384	24,748	46,870	538	None.	96,540
Massachusetts	95,453	87,289	190,582	5,463	None.	378,787
Rhode Island	16,019	15,799	32,652	3,407	948	68,825
Connecticut	60,523	54,403	117,448	2,808	2,764	237,946
New York	83,700	78,122	152,320	4,654	21,324	340,120
New Jersey	45,251	41,416	83,287	2,762	11,423	184,139
Pennsylvania	110,788	106,948	206,363	6,537	3,737	434,373
Delaware	11,783	12,143	22,384	3,899	8,887	[2] 59,094
Maryland	55,915	51,339	101,395	8,043	103,036	319,728
Virginia	110,936	116,135	215,046	12,866	292,627	747,610
Kentucky	15,154	17,057	28,922	114	12,430	73,677
North Carolina	69,988	77,506	140,710	4,975	100,572	393,751
South Carolina	35,576	37,722	66,880	1,901	107,094	249,073
Georgia	13,103	14,044	25,739	398	29,264	82,548
Total number of inhabitants of the United States exclusive of S. Western and N. territory	807,094	791,850	1,541,263	59,150	694,280	3,893,635

	Free white males of 21 years and upward.	Free males under 21 years of age.	Free white females.	All other persons.	Slaves.	Total.
S. W. territory	6,271	10,277	15,365	361	3,417	35,691
N. "						

[1] The census of 1790, published in 1791, reports 16 slaves in Vermont. Subsequently, and up to 1860, the number is given as 17. An examination of the original manuscript returns shows that there never were any slaves in Vermont. The original error occurred in preparing the results for publication, when 16 persons, returned as "Free colored," were classified as "Slave."

[2] Corrected figures are 85,425, or 114 less than figures published in 1790, due to an error of addition in the returns for each of the towns of Fairfield, Milton, Shelburne, and Williston, in the county of Chittenden; Brookfield, Newbury, Randolph, and Strafford, in the county of Orange; Castleton, Clarendon, Hubbardton, Poultney, Rutland, Shrewsbury, and Wallingford, in the county of Rutland; Dummerston, Guilford, Halifax, and Westminster, in the county of Windham; and Woodstock, in the county of Windsor.

[3] Corrected figures are 59,096, or 2 more than figures published in 1790, due to error in addition.

This table is from "Heads of Families at the First Census of the United States Taken in the Year 1790, Massachusetts, Washington, DC: Department of Commerce and Labor, Bureau of the Census, 1908.

The Census and Reapportionment

Overview

In this lesson students become familiar with two of the methods that have been proposed or actually used to reapportion the 435 seats in the House of Representatives after each decennial census. The methods use only simple mathematics but illustrate two points: there can be different mathematical techniques from which to choose, and simple arithmetic is used in making important decisions.

Mathematical Concepts

different methods of reapportionment

Skills

working with integers and decimals, rounding, proportions, percentages, exponents

Vocabulary

reapportionment, average district size, integers

Materials

copies of Table A, "Representatives under Each Apportionment"; Table B, "1990 Population by State and Numbers of Representatives"; Handouts 2-2 through 2-6; calculators. Optional are computers and spreadsheet software, particularly for Activity 2, but also for Activities 4, 5, 6, and 7.

Time Needed

five–nine class periods

Activity 1: Background

Use Tables A and B

State populations have changed enormously over time, and therefore so have the number of seats they are allotted in the House of Representatives. Students can look at Table A and observe many trends and interesting phenomena.

Give each small group of students a copy of Table A. Have each group study the table and write ten sentences about the information presented.

Questions to Encourage Observations and Discussion

- Why did some states begin to have representatives later than other states?
- How many representatives did your state have in 1930 and in 1960? Has the current number of representatives from your state changed from what the number was in 1930? in 1960? (Current numbers are listed in Table B.) If your state has been relatively stable, look at California and Florida.
- Which states have continually increased their representation in the House? Which states have lost seats in recent decades?
- Pennsylvania has lost seats in the House in every reapportionment since 1930. Does this necessarily mean that Pennsylvania's population has decreased?

Help students understand that the answer to the last question is no. The population of Pennsylvania has increased, but its population has increased more slowly than the population of other states. Thus, its relative proportion of seats in the House has decreased. You can also use Table B to illustrate this for the period 1980–90.

You may also want to have students experiment with hypothetical situations and make-believe numbers in order to understand the last question. Have them first state what they think is true (whether the answer is yes or no), and then let them test their conjectures in several hypothetical situations. You may want to pose the situations yourself or have the students invent them.

Now that students have seen the number of representatives change quite a lot over time, they may be wondering how representation is determined.

Have each group of students brainstorm and invent a method for calculating the number of seats per state. They may invent various approaches. Have them write a clear explanation of their methods and share their ideas with the rest of the class. The goal of this exercise is to have students experiment with the data and to think and communicate clearly about a mathematical procedure and its application.

Activity 2: Average District Size

Use Student Handout 2-2 and Table B

The activity on Handout 2-2 asks students to compare the district sizes of each state, based on the reapportionment following the 1990 census. The goal is for students to assess the accuracy of the statement, "Roughly, each state receives the proportion of 435 total representatives that the state's population makes up of the total U.S. population." Note that this can vary from state to state. New York, New Jersey, and Massachusetts each have fewer representatives than this statement would imply.

Have students do this activity in groups of three to four. They will be able to divide the computing labor among group members.

If computers and spreadsheet software are available, have students do this activity with these tools. The spreadsheet they create will be usefull for Activities 2, 4, 5, 6, and 7. They can enter the data they think they will need from Table B (they might not need all the data listed). Students can then expand their spreadsheets to give them new information. For example, they can create columns for the number of residents per representative, they can sort for certain states, etc.

Activity 3: Reapportioning in the Country of Frachat

Use Student Handout 2-3

This activity is designed to give students an opportunity to play with the concept of reapportioning representatives in a make-believe situation. Students will then have first-hand experience in the complex task of balancing divergent populations in a fair and equal way.

Divide students into pairs or groups of three. To start, students will place coins in the appropriate boxes on the map of Frachat to indicate the initial assignment of representatives.

The chart "Population Change in the Country of Frachat" shows how the population of Frachat has changed. Based on this, students will move the coins around as they begin to decide how to allot each state its representatives. They may choose to place the coins in several different configurations before deciding which plan makes the most sense to them. Handout 2-3 gives directions and questions for students to contemplate as they become involved in this activity.

Upon completion, have the students present their findings and methods orally and in writing. Generate a discussion on the methods used and reasons for making these choices.

Answers to questions from Handout 2-3:

2. Twenty-one coin representatives.
3. 2,100,000 people live in this country.
5. Methods and responses may vary.

Activity 4: Reapportioning by the Webster Method

Use Student Handout 2-4 and Table B

In this and the following activity students investigate two different methods for reapportioning the seats in the House. You may decide to have students experiment with only one method, depending on time and inclination.

Handout 2-4 outlines the process for calculating via the Webster method. Give students the handout and Table B. The handout is structured to allow students to experiment. Encourage them to devise a system for organizing the many calculations they will have to perform. Students can use spreadsheet software and computers for this activity and for activities 5, 6, and 7, if these resources are available.

Summary of the Webster Method (also called the Major Fractions Method)[1]

1. In this method the total U.S. population is first divided by 435 to determine the *average district size.* Call this number S. (This is really just a preliminary guess; in the process this number will be adjusted upward or downward.)

2. To determine the number of House seats that will go to a given state, divide that state's population by S. This will give a decimal number. Round this number to the nearest integer (round up if the fractional part is .5 or bigger and down otherwise). This is a preliminary guess for that state. (If this number rounds to zero for some state, that state is still given 1 representative; but this doesn't happen with the current population figures.) Do this for each state.

3. Count how many total seats have been assigned. At this point it may happen that either more than 435 or fewer than 435 seats have been assigned. But we have to assign *exactly* 435 states, and, to be fair, the method has to be the same for all states. The Webster method proceeds as follows.

4. Adjust the number S either up or down. If you have assigned too few seats, adjust S down. If you have assigned too many seats, adjust S up. Then go through step 2 again with the new value of S. If you have the right number of total seats, stop. If not, readjust S and go to step 2 again.

Repeat 2, 3, and 4 until you have assigned the correct number of seats.

---

[1] See David C. Huckabee, "Apportioning Seats in The House of Representatives: the Method of Equal Proportions," Congressional Research Service Report for Congress, 88-143 BOV, Library of Congress, February 1988.

Example

This is one of the examples the students will work out in Handout 2-4. Note: *Do not* use this example to explain the method to the students. Let them hammer it out for themselves. Give them hints so they stay on the right track.

Say our country just consists of the eight states whose names begin with M, and that the House of Representatives in the country of M-States has 59 seats (the actual number the M-States have after the 1990 Census).

1. From Table B the total population of these states is 34,304,611. The average district size S is therefore 581,434 (total population divided by the number of seats).

2. Dividing each state's population by S, we get the following figures, which represent the number of districts in each state:

Michigan	16.044	rounds to	16
Massachusetts	10.369	"	10
Missouri	8.836	"	9
Maryland	8.253	"	8
Minnesota	7.545	"	8
Mississippi	4.448	"	4
Maine	2.121	"	2
Montana	1.382	"	1

3, 4. The total number of seats assigned is 58. We missed one! Now adjust S down a little. Let's try 575,000 for S.

Doing step 2 again gives:

Michigan	16.224	rounds to	16
Massachusetts	10.485	"	10
Missouri	8.935	"	9
Maryland	8.345	"	8
Minnesota	7.630	"	8
Mississippi	4.498	"	4
Maine	2.145	"	2
Montana	1.398	"	1

We're still off by one! It looks like Mississippi and Massachusetts are both close to getting another representative. If we adjust S down by just a little more, presumably Mississippi will "go over the top" (past .5) but Massachusetts will not.

By replacing S with 573,000, both states go over the .5 limit, so we decreased S by too much!

This time let's try S = 574,500. Doing the calculation we find that Mississippi goes over the top but Massachusetts does not. The correct numbers of representatives for each state are these:

Michigan	16
Massachusetts	10
Missouri	9
Maryland	8
Minnesota	8
Mississippi	5
Maine	2
Montana	1

These figures happen to agree with the figures in Table B. This will not necessarily be the case for an arbitrary subset of the states, since the correct figures depend on doing the calculation for all fifty states together.

Activity 5: Reapportioning by the Hamilton-Vinton Method

Use Student Handout 2-5 and Table B

Give students Handout 2-5 and Table B. The handout walks the students through the Hamilton-Vinton reapportionment method.

This method was proposed in 1981 but was not adopted.[2] It is simpler to use than the Webster method, but can sometimes lead to paradoxical results.

1. Find the average district size, as in the Webster method.

2. Divide each state's population by the average district size and assign seats equal to the whole number part of the quotient. At this point do not do any rounding as in the Webster method. (The integer part of the quotient x is also called the "greatest integer" in x, or the "floor" of x.) Do not throw away the fractions! (Note: If the greatest integer is zero, assign that state one seat, and do not keep the fractional part in 3 below.)

3. Order the fractional parts from largest to smallest.

4. If all the required seats were assigned in 2, there is nothing more to do. If not, assign one seat in turn to the states in order of largest fractional part, until the required number of seats has been assigned. The following example gives the answer to problem 2 on the handout.

Example

Let's take the same example as above.

---

[2] Ibid, p.13.

Michigan	16.044	gives	16
Massachusetts	10.369	"	10
Missouri	8.836	"	8
Maryland	8.253	"	8
Minnesota	7.545	"	7
Mississippi	4.448	"	4
Maine	2.121	"	2
Montana	1.382	"	1

After assigning the whole numbers, we have three seats left to assign. In order from largest fraction to smallest we have:

Missouri	.836
Minnesota	.545
Mississippi	.448
Montana	.382
Massachusetts	.369
Maryland	.253
Maine	.121
Michigan	.044

Thus the three remaining seats go to Missouri, Minnesota, and Mississippi, and we get the same answer as we did before.

Activity 6: Comparing the Two Methods

Use Student Handout 2-5 and Table B

The section titled "Peculiarities of the Hamilton-Vinton Method" in Handout 2-5 presents a situation in which the two methods described above do not lead to the same result. The first part of this exercise asks students to compute what the population of the five fastest-growing states will be in 2010, using the percentages listed in Table B, which give net growth rates from 1980 to 1990. Using these new population figures the students apply both the Webster method and the Hamilton-Vinton method to reapportion the seats in the House (for just these five states) in the year 2010.

The peculiarity of the Hamilton-Vinton method demonstrated in these exercises is that states can sometimes gain extra representatives at the expense of other states that have a higher percentage rate of growth. This is the reason the "method of equal proportions" is used instead in the actual reapportionment.

The Webster and Hamilton-Vinton methods do lead to the same answer in 5 and 6, but not in 7 and 8. The answer to 9 is Florida.

Prior to having students compute the populations in 2010 of the five fastest-growing states, you may want to go through the following activity with them.

Using Percentage Growth to Predict Population Change

Review with students how to project the population in 2010, as follows.

By way of an example, assume the population of state X in 1980 is 20,000,000, and its rate of growth from 1980 to 1990 was 11.5%. Ask the students:

- What does the 11.5% figure mean? (The difference between the population of state X in 1990 and 1980 is 11.5% of the population in 1980.)
- Therefore, how can you compute the difference between the population of state X in 1980 and 1990? (Multiply .115 by 20,000,000. You get 2,300,000.)
- So what is the population of state X in 1990? (Just add: 20,000,000 + .115 (20,000,000) = 22,300,000.)

Now we know the population in 1990. We want to find the population in 2010. We will assume the same rate of growth continues between 1990 and 2010.

- How should we do it?

Help students see they can apply the same reasoning to each of the decades 1990–2000 and 2000–2010.

Using the figure 22,300,000 to compute the population in 2000 gives

$$22{,}300{,}000 + .115\,(22{,}300{,}000) = 24{,}864{,}500,$$

and using the last figure to compute the population in 2010 gives

$$24{,}864{,}500 + .115\,(24{,}864{,}500) = 27{,}723{,}917.5\,,$$

which we should round to 27,723,917.

Ask students if they notice any patterns in these computations. They all have the form

$$P + .115\,(P).$$

- Is there any way to simplify this calculation? (Use the distributive law. If they don't see this, ask them to do the computation without any addition.)

Conclusion: we can write $P + .115\,(P) = (1 + .115) \cdot P = 1.115 \cdot P$, or just $1.115P$ for short. In each of the above calculations we could have just multiplied by 1.115. (This is one of the values of algebra, that it lets us express many calculations using one mathematical "sentence" or formula.)

Now the students will be able to calculate without difficulty the population in 2010 for each of the five fastest-growing states. (Note that Table B gives the population of each

state in 1990, not in 1980.) You can take this calculation further by giving students the following challenge.

Formula-Making Activity

1. Find a formula for the population of state X in 2010, if you know its population is P in 1990 and its rate of growth is 11.5% every decade. What would the formula be if 11.5% were replaced by 13%? If 11.5% were replaced by r%?

2. If state X has population P in 1990, and its rate of growth is 11.5 % every decade, find a formula for the population of state X exactly three decades after 1990 (in 2020).

3. If state X has population P in 1990, and its rate of growth is 11.5 % every decade, find a formula for the population of state X exactly four decades after 1990 (in 2030).

4. Do you notice a pattern in 2 and 3? If state X has population P in 1990, and its rate of growth is 11.5% every decade, find a formula for the population of state X exactly k decades after 1990.

5. What would your formulas be in 2–4 if 11.5% were replaced by 13%? If 11.5% were replaced by r%?

Note: To predict what the population of state X (with a population of P people in 1990 and an 11.5% rate of growth) will be in k decades, students may come up with the formula

$$\text{population k decades after 1990} = (1.115)^k * P.$$

If they find more complicated formulas, encourage them to simplify their expressions using the distributive law, as in the above discussion, or using exponents.

The general formula would be

$$\text{population k decades after 1990} = (1 + r/100)^k * P,$$

if the rate of growth every decade is r%.

This exercise gives a natural opportunity to bring in exponents.

Going Further with the Webster Method

The method actually used by Congress is a modification of the Webster method called the *"method of equal proportions."*[3] There are two main differences between the Webster method and the method of equal proportions.

Priority Lists

The first difference between our description of the Webster method and the way the method is applied in practice is that the repeated adjustments of the average district size are avoided. This is done by ranking each state's claim to each representative in a "priority list."

Briefly, the state's population is divided by the rounding point (N+.5) between N and N+1 to determine that state's claim on N+1 representatives.

For example, Michigan's "claim" to fifteen representatives is its population divided by 14.5, or 643,364.41. This is the largest district size under which Michigan would have at least fifteen seats in the House. Michigan's "claim" to sixteen seats is its population divided by 15.5, or 601,857.03, again, the largest district size that would guarantee Michigan at least sixteen seats.

Performing this calculation for each of the eight M-states (for numbers of seats relatively close to the numbers they actually receive) and rank ordering their claims from highest to smallest gives the following priority list.

Seat	District Size	State - Number of Representatives
41	822,148	Maine-2
42	811,198	Michigan-12
43	803,873	Massachusetts-8
44	797,641	Minnesota-6
45	790,431	Missouri-7
46	746,302	Michigan-13
47	738,983	Mississippi-4
48	738,249	Maryland-7
49	709,300	Massachusetts-9
50	691,021	Michigan-14
51	685,040	Missouri-8
52	674,927	Minnesota-7
53	643,364	Michigan-15
54	639,816	Maryland-8
55	634,636	Massachusetts-10
56	604,447	Missouri-9
57	601,857	Michigan-16
58	584,937	Minnesota-8

[3] Ibid.

59	574,765	Mississippi-5
60	574,195	Massachusetts-11
61	565,380	Michigan-17
62	564,543	Maryland-9
63	540,821	Missouri-10
64	535,770	Montana-2
65	533,073	Michigan-18
66	524,265	Massachusetts-12
67	516,121	Minnesota-9
68	505,118	Maryland-10
69	504,258	Michigan-19

This is just a partial list: the only district sizes included are those between 504,258 and 822,148. The full list would start with seat 9 (Michigan-2 and district size 6,219,189) and include every state's claim to every seat, going as far as necessary to guarantee the right number of seats.

The above list gives the order in which House seats are assigned. At the start of the list, the following seats have already been assigned:

Michigan-11
Massachusetts-7
Missouri-6
Maryland-6
Minnesota-5
Mississippi-3
Maine-1
Montana-1

Thus the first seat in the partial list is seat 41. To assign fifty-nine seats count down the list until you reach the fifty-ninth seat, which is Mississippi-5. The last listing of each state above this position gives the correct number of representatives for that state.

To assign sixty-five seats instead of fifty-nine seats to these eight states, simply continue counting down the list to seat 65, which is Michigan-18. Thus Michigan would have eighteen seats, Montana two, Missouri ten, Maryland nine, Massachusetts eleven, Mississippi five, Minnesota eight, and Maine two.

Note that once this priority list is generated, no extra calculations are required. Only one (albeit large) calculation needs to be performed, and then the results can be read off the priority list, no matter what number of House seats are to be assigned. This method is described in detail in Huckabee (See footnote 1 of this section).

The priority list always leads to the same answer given by the Webster method explained above, but is more efficient. It also shows some interesting things. Notice how close the rankings are for Mississippi-5 and Massachusetts-11. This shows why we had to take our value of S in the above calculation to be 574,500.

Geometric Means

The second main difference between the Webster method and the method of equal proportions is that the equal proportions method rounds fractions using the geometric mean.

The *geometric* mean of two numbers a and b is the number $\sqrt{(ab)}$. This is analogous to the *arithmetic* mean, or average, of a and b, which is $(a+b)/2$.

The numbers a, $(a+b)/2$, b are in arithmetic progression, which means that you add the same thing to go from a to $(a+b)/2$ that you do to go from $(a+b)/2$ to b.

In a similar way, a, $\sqrt{(ab)}$, and b are in geometric progression, which means that you *multiply* the same thing to go from a to $\sqrt{ab}$ that you do to go from $\sqrt{ab}$ to b.

For example, the geometric mean of 2 and 8 is $\sqrt{16} = 4$, so the numbers 2, 4, 8 are in geometric progression (since you multiply each number by 2 to get the next one).

This works even if the numbers do not work out evenly: the geometric mean of 3 and 4 is $\sqrt{12} = 3.464101\ldots$, and the numbers 3, 3.464101. . ., 4 are in geometric progression (you multiply each number by $(\sqrt{12})/3 = 1.547005\ldots$ to get the next one; try it!).

Activities on the Geometric Mean

1. Compute the geometric mean of the numbers 1 and 2, 2 and 3, 3 and 4, etc., all the way up to the geometric mean of 29 and 30. Graph your results. Do you notice any patterns or trends?

2. Is the following statement true or false: the geometric mean of two numbers is always *less than* their arithmetic mean?

3. Say why the geometric mean of a and b always lies *between* a and b.

4. What number x solves the proportion $a/x = x/b$?

Now back to reapportionment. The method of equal proportions uses the geometric mean of consecutive integers instead of the arithmetic mean to round fractions.

For example, instead of rounding numbers up at 4.5 and down below 4.5, the geometric mean of 4 and 5 is used instead: this is $\sqrt{(4 * 5)} = \sqrt{20} = 4.4721359\ldots$. Thus fractions are rounded up at 4.47. . . instead of at 4.5. The number 4.48 would be rounded up to 5, but the number 4.35 would be rounded down to 4.

If you are between 10 and 11, the rounding point would be at $\sqrt{(10 * 11)} = \sqrt{110} = 10.488\ldots$. The number 10.42 would be rounded down to 10, while 10.491 would be rounded up to 11.

Using the geometric mean, the rounding point depends on which two integers the number you are rounding lies between. This gives a slight advantage to smaller states. Since the geometric mean gives a lower rounding point for smaller integers, smaller states do not have to have as large a fraction to be awarded an extra representative.

The priority list explained above can also be used with geometric means. This is the way the method of equal proportions is actually applied in practice.

Thus to compute Michigan's claim to fifteen representatives, instead of dividing its population by 14.5, divide by the geometric mean of 14 and 15, or 14.49137. This gives the district size 643,747. (Compare this with 643,364, obtained by dividing by 14.5.)

Activities on Equal Proportions and Priority Lists

1. Recreate the above priority list for the M-states using geometric means instead of arithmetic means. What are the differences between the two lists?

2. Try to understand why the above priority ranking works and explain it to the class. Give an example using a selection of states from Table B.

Ideas for Other Topics

Most of the time the differences between the Webster method and the method of equal proportions are very subtle, which is why we have chosen to present directly to students only the simpler method using arithmetic means.

If you would like to give the students further challenges, however, geometric means can be used to motivate several interesting topics, including functions (study the geometric mean of 1 and x, as x varies), limits (what is the behavior of the geometric mean of N and N+1, as N gets larger?), and simple algebraic proofs (see problems 2 and 3 in "Activities for the Geometric Mean" above).

Activity 7: The Effects of Undercounting on Reapportionment

Use Student Handout 2-6

The last activity gives students an idea of the possible effects on reapportionment of an undercount. We chose six states affected by the undercount in the 1990 census: if the undercount had been adjusted, California and Arizona would each have gained a seat, and Pennsylvania and Wisconsin would each have lost a seat. Illinois and Missouri are included because they offer contrast to Pennsylvania and Wisconsin.

The first exercise on the handout shows that an undercount may have no effect, if it is distributed uniformly over different states. In this exercise the 2.1 percent increase does not change representation.

The second exercise shows that if the undercount in some states is disproportionately large, it can have a big effect on the assignment of seats in the House. In this exercise California and Arizona each gain one seat; Pennsylvania and Wisconsin each lose one.

Table A

Representatives under each Apportionment (1790–1980)

State	Constitutional apportionment	First Census, 1790	Second Census, 1800	Third Census, 1810	Fourth Census, 1820	Fifth Census, 1830	Sixth Census, 1840	Seventh Census, 1850	Eighth Census, 1860	Ninth Census, 1870	Tenth Census, 1880	Eleventh Census, 1890	Twelfth Census, 1900	Thirteenth Census, 1910 [1]	Fifteenth Census, 1930	Sixteenth Census, 1940	Seventeenth Census, 1950	Eighteenth Census, 1960	Nineteenth Census, 1970	Twentieth Census, 1980
Alabama				[2] 1	3	5	7	7	6	8	8	9	9	10	9	9	9	8	7	7
Alaska																	[2] 1	1	1	1
Arizona														[2] 1	1	2	2	3	4	5
Arkansas						[2] 1	1	2	3	4	5	6	7	7	7	7	6	4	4	4
California							[2] 2	2	3	4	6	7	8	11	20	23	30	38	43	45
Colorado										[2] 1	1	2	3	4	4	4	4	4	5	6
Connecticut	5	7	7	7	6	6	4	4	4	4	4	4	5	5	6	6	6	6	6	6
Delaware	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Florida							[2] 1	1	1	2	2	2	3	4	5	6	8	12	15	19
Georgia	3	2	4	6	7	9	8	8	7	9	10	11	11	12	10	10	10	10	10	10
Hawaii																	[2] 1	2	2	2
Idaho											[2] 1	1	1	2	2	2	2	2	2	2
Illinois				[2] 1	1	3	7	9	14	19	20	22	25	27	27	26	25	24	24	22
Indiana				[2] 1	3	7	10	11	11	13	13	13	13	13	12	11	11	11	11	10
Iowa							[2] 2	2	6	9	11	11	11	11	9	8	8	7	6	6
Kansas									1	3	7	8	8	8	7	6	6	5	5	5
Kentucky		2	6	10	12	13	10	10	9	10	11	11	11	11	9	9	8	7	7	7
Louisiana				[2] 1	3	3	4	4	5	6	6	6	7	8	8	8	8	8	8	8
Maine				[3] 7	7	8	7	6	5	5	4	4	4	4	3	3	3	2	2	2
Maryland	6	8	9	9	9	8	6	6	5	6	6	6	6	6	6	6	7	8	8	8
Massachusetts	8	14	17	[3] 13	13	12	10	11	10	11	12	13	14	16	15	14	14	12	12	11
Michigan						[2] 1	3	4	6	9	11	12	12	13	17	17	18	19	19	18
Minnesota								[2] 2	2	3	5	7	9	10	9	9	9	8	8	8
Mississippi				[2] 1	1	2	4	5	5	6	7	7	8	8	7	7	6	5	5	5
Missouri					1	2	5	7	9	13	14	15	16	16	13	13	11	10	10	9
Montana											[2] 1	1	1	2	2	2	2	2	2	2
Nebraska									[2] 1	1	3	6	6	6	5	4	4	3	3	3
Nevada									[2] 1	1	1	1	1	1	1	1	1	1	1	2
New Hampshire	3	4	5	6	6	5	4	3	3	3	2	2	2	2	2	2	2	2	2	2
New Jersey	4	5	6	6	6	6	5	5	5	7	7	8	10	12	14	14	14	15	15	14
New Mexico														[2] 1	1	2	2	2	2	3
New York	6	10	17	27	34	40	34	33	31	33	34	34	37	43	45	45	43	41	39	34
North Carolina	5	10	12	13	13	13	9	8	7	8	9	9	10	10	11	12	12	11	11	11
North Dakota											[2] 1	1	2	3	2	2	2	2	1	1
Ohio			[2] 1	6	14	19	21	21	19	20	21	21	21	22	24	23	23	24	23	21
Oklahoma													[2] 5	8	9	8	6	6	6	6
Oregon								[2] 1	1	1	1	2	2	3	3	4	4	4	4	5
Pennsylvania	8	13	18	23	26	28	24	25	24	27	28	30	32	36	34	33	30	27	25	23
Rhode Island	1	2	2	2	2	2	2	2	2	2	2	2	2	3	2	2	2	2	2	2
South Carolina	5	6	8	9	9	9	7	6	4	5	7	7	7	7	6	6	6	6	6	6
South Dakota											[2] 2	2	2	3	2	2	2	2	2	1
Tennessee		[2] 1	3	6	9	13	11	10	8	10	10	10	10	10	9	10	9	9	8	9
Texas							[2] 2	2	4	6	11	13	16	18	21	21	22	23	24	27
Utah											[2] 1	1	2	2	2	2	2	2	2	3
Vermont		2	4	6	5	5	4	3	3	3	2	2	2	2	1	1	1	1	1	1
Virginia	10	19	22	23	22	21	15	13	11	9	10	10	10	10	9	9	10	10	10	10
Washington											[2] 1	2	3	5	6	6	7	7	7	8
West Virginia										3	4	4	5	6	6	6	6	5	4	4
Wisconsin							[2] 3	3	6	8	9	10	11	11	10	10	10	10	9	9
Wyoming											[2] 1	1	1	1	1	1	1	1	1	1
Total	65	106	142	186	213	242	232	237	243	293	332	357	391	435	435	435	437	435	435	435

[1] No apportionment was made in 1920.

[2] The following representation was added after the several census apportionments indicated when new States were admitted and is included in the above table:

First. Tennessee, 1.
Second. Ohio, 1.
Third. Alabama, 1; Illinois, 1; Indiana, 1; Louisiana, 1; Mississippi, 1.
Fifth. Arkansas, 1; Michigan, 1.
Sixth. California, 2; Florida, 1; Iowa, 2; Texas, 2; Wisconsin, 2.
Seventh. Minnesota, 2; Oregon, 1.
Eighth. Nebraska, 1; Nevada, 1.
Ninth. Colorado, 1.
Tenth. Idaho, 1; Montana, 1; North Dakota, 1; South Dakota, 2; Washington, 1; Wyoming, 1.
Eleventh. Utah, 1.
Twelfth. Oklahoma, 5.
Thirteenth. Arizona, 1; New Mexico, 1.
Seventeenth. Alaska, 1; Hawaii, 1.

[3] Twenty Representatives were assigned to Massachusetts, but 7 of them were credited to Maine when that area became a State.

Table B

1990 Population by State and Numbers of Representatives

State	Population	Percent change 1980–90	Number of U.S. Reps.	Change since 1980
California	29,839,250	25.7	52	+7
New York	18,044,505	2.5	31	-3
Texas	17,059,805	19.4	30	+3
Florida	13,003,362	32.7	23	+4
Pennsylvania	11,924,710	0.1	21	-2
Illinois	11,466,682	0.0	20	-2
Ohio	10,887,325	0.5	19	-2
Michigan	9,328,784	0.4	16	-2
New Jersey	7,748,634	5.0	13	-1
North Carolina	6,657,630	12.7	12	+1
Georgia	6,508,419	18.6	11	+1
Virginia	6,216,568	15.7	11	+1
Massachusetts	6,029,051	4.9	10	-1
Indiana	5,564,228	1.0	10	—
Missouri	5,137,804	4.1	9	—
Wisconsin	4,906,745	4.0	9	—
Tennessee	4,896,641	6.2	9	—
Washington	4,887,941	17.8	9	+1

Data supplied by the Congressional Research Service of the Library of Congress.

Table B (continued)

Maryland	4,798,622	13.4	8	—
Minnesota	4,387,029	7.3	8	—
Louisiana	4,238,216	0.3	7	-1
Alabama	4,062,608	3.8	7	—
Kentucky	3,698,969	0.7	6	-1
Arizona	3,677,985	34.8	6	+1
South Carolina	3,505,707	11.7	6	—
Colorado	3,307,912	14.0	6	—
Connecticut	3,295,669	5.8	6	—
Oklahoma	3,157,604	4.0	6	—
Oregon	2,853,733	7.9	5	—
Iowa	2,787,424	-4.7	5	-1
Mississippi	2,586,443	2.1	5	—
Kansas	2,485,600	4.8	4	-1
Arkansas	2,362,239	2.8	4	—
West Virginia	1,801,625	-8.0	3	-1
Utah	1,727,784	17.9	3	—
Nebraska	1,584,617	0.5	3	—
New Mexico	1,521,779	16.3	3	—
Maine	1,233,223	9.2	2	—
Nevada	1,206,152	50.1	2	—
Hawaii	1,115,274	14.9	2	—
New Hampshire	1,113,915	20.5	2	—

Table B (continued)

Idaho	1,011,986	6.7	2	—
Rhode Island	1,005,984	5.9	2	—
Montana	803,655	1.6	1	-1
South Dakota	699,999	0.8	1	—
Delaware	668,696	12.1	1	—
North Dakota	641,364	-2.1	1	—
Vermont	564,964	10.0	1	—
Alaska	551,947	36.9	1	—
Wyoming	455,975	-3.4	1	—

Student Handout 2-2

Reapportionment of U.S. Representatives

One of the important uses for census data is determining the number of representatives that each state is allotted in the U.S. House of Representatives. The number of representatives a state has is based on that state's population. Since the population is constantly in flux, the U.S. Constitution requires that the number of representatives from each of the states be recomputed every ten years. This process is called *reapportionment* and it is performed immediately following the completion of the census.

The Constitution requires that each state have at least one representative and that there be no more than one representative for every 30,000 people. Since 1911 the total number of representatives in the House has been fixed at 435.[4] Roughly, each state receives the proportion of 435 total representatives that the state's population makes up of the total U.S. population. But this statement is really only approximately true. Congress has used several different methods to do the reapportioning. We'll investigate some of these methods in the activities that follow.

After the reapportionment following the 1990 census there is (as of January 1993) one U.S. representative for approximately every 570,000 people.[5] In each state, therefore, roughly 570,000 people will make up one voting district. But this number will vary greatly from state to state.

Beginning Activity: Average District Sizes

To see exactly how the number of people per voting district varies from state to state, use Table B. This table gives the population of each state, the number of representatives that each state was allotted after the 1990 census, and the change in the number of representatives from the 1980 figures.

1. Determine the average population of a voting district of each state.

2. Which state has the largest district size? Which has the smallest?

---

[4] David C. Huckabee, "Apportioning Seats in the House of Representatives: The Method of Equal Proportions," Congressional Research Service Report for Congress, 88–143 BOV, Library of Congress, February 1988, p. 6.

[5] Ibid.

3. Determine the average district size for the whole country.

4. Is there more than one way to do the last computation? Why or why not?

5. What percentage of the total population is contained in each of the five largest and five smallest states?

6. What proportion of representatives do each of the five largest and five smallest states have?

7. Using your answers to questions 5 and 6, say whether you think the statement is true that "Roughly, each state receives the proportion of 435 total representatives that the state's population makes up of the total U.S. population." Why?

8. For each of the states New York, New Jersey, and Massachusetts, find the relative proportion of the state population to the total U.S. population. If these states had a proportion of the 435 representatives equal to the population proportion, how many representatives would each state have? Compare with the actual number of representatives each state has. If you were from one of these states, what case could you make for having the greater number of representatives?

Student Handout 2-3

Classroom Simulation: The Reapportionment of the Country of Frachat

In this activity you will play with different possibilities for deciding how many representatives (coins) a given state should receive. The directions and questions listed below will help you focus on some of these possibilities.

1. Place the coins in the appropriate boxes on the map of Frachat. The chart indicates the number of coins to be placed in each state. Each coin represents 100,000 people.

2. How many coin representatives are there in the country of Frachat?

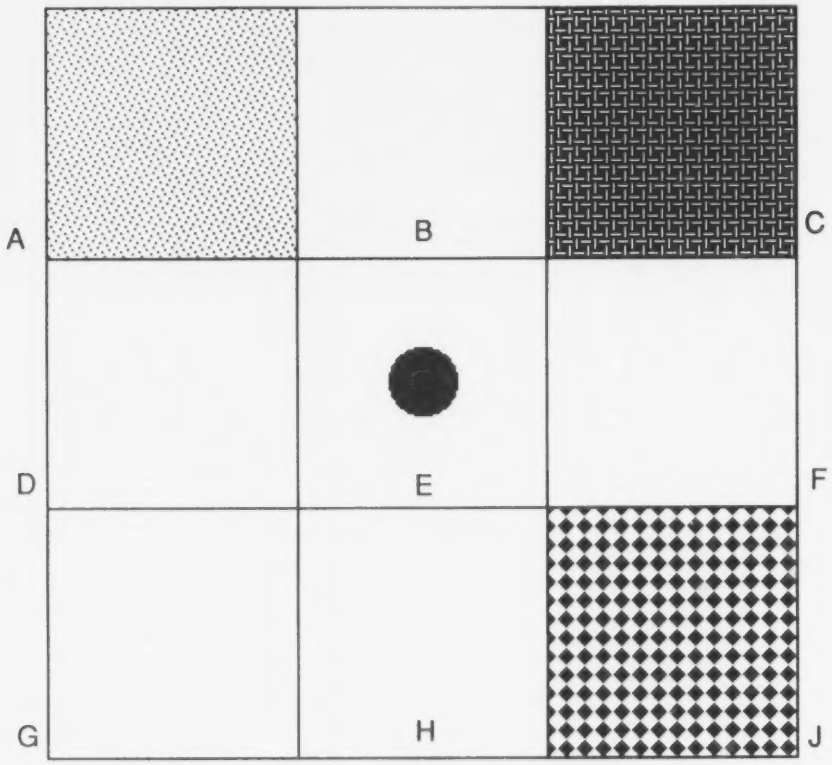
3. How many people live in the entire country?

4. Now look at the chart entitled "Population Change in the Country of Frachat." If the population of Frachat changes in this way, how would you now arrange the coins?

   When working on this activity please note the *two* restrictions: The coins must still total *twenty-one,* and each state must be represented by at least one coin.

5. What method did you use for reapportioning your coin representatives? Explain why you chose this method.

The Reapportionment of the Country of Frachat



Each box represents a state in the country of Frachat

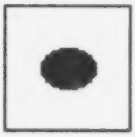
Key: Initial Representation per State

Each coin represents 100,000 people

State					
Number of Coins	2	5	3	1	6

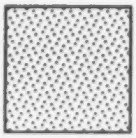
Population Change in the Country of Frachat

Increase of 125,000

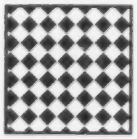


Increase of 219,000 people

Increase of 58,000 people for each state



Increase of 80,000 people



No increase or decrease

Student Handout 2-4

Reapportioning by the Webster Method

In this activity you will perform the reapportionment yourselves. Use Table B. If you are using a calculator, be careful entering numbers and recheck your computations.

1. The N-States

   To simplify matters a little bit, we'll just look at the eight states whose names begin with the letter N. We'll pretend that these states make up their own country and call them N-States.

   Calculate the total population of the N-States. Figure out the what the average district size should be, if the N-House of Representatives contains 67 seats.

   Use the average district size to determine how many districts each state receives. If decimals or fractions appear in your computations, round up to the next whole number when the decimal is .5 or greater. (For example 4.506788123 would round to 5, while 4.49930221 would round to 4.)

2. The M-States

   Now perform the same calculation for the M-States, the eight states whose names begin with M, and assume that the M-House of Representatives has 59 seats. Use the same method you used in 1.

   Are you able to assign all 59 representatives? If not, adjust the average district size upward or downward and try the computation again.

   For example, if your average district size is 570,000 and your computation does not assign the right number of representatives, arbitrarily try a slightly smaller or larger district size, say 580,000. Determine whether you are getting closer or are further off the mark. If it doesn't work with the new district size, try decreasing instead or increasing even more. Continue trying different numbers until you find one that works.

3. The Fastest-Growing States

   Use the same method you used in 2 to assign 84 seats to the five fastest-growing states (from 1980 to 1990): California, Florida, Nevada, Alaska, Arizona.

The method you used in the above exercise is called the Webster method, because it was proposed by Daniel Webster in 1832.[6] It is also called the "major fractions method." Though it has been used in reapportioning, it is not the method in current use. The method used now is a modification of the Webster method: Instead of rounding up at fractions equal to or greater than .5, it uses the concept of geometric means to do the rounding. This method is called the "method of equal proportion".

Summary of the Webster Method

1. First divide the total population by the desired number of House representatives (call it R), to determine the *average district size*. Call this number S. (This is really just a preliminary guess; in the process this number will be adjusted upward or downward.) Thus Population/R = S.

2. To determine the number of House seats that will go to a given state, divide that state's population by S. This will give a decimal number. Round this number to the nearest integer (round up if the fractional part is .5 or bigger and down otherwise). This is a preliminary guess for that state. (If this number rounds to zero for some state, that state is still given one representative.) Do this for each state.

3. Count how many total seats have been assigned. At this point it may happen that either more than R or fewer than R seats have been assigned.

4. Adjust the number S either up or down. If you have assigned too few seats, adjust S down. If you have assigned too many seats, adjust S up. Then go through 2 again with the new value of S. If you have the right number of total seats, stop. If not, readjust S and go to 2 again.

   Repeat 2, 3, and 4 until you have assigned the correct number of seats.

---

[6] Ibid, p.13.

Student Handout 2-5

The Hamilton-Vinton Method

In 1981 a new method called the Hamilton-Vinton method was proposed in a bill before Congress but was not adopted. This method works as follows.

1. First, each state is assigned one representative.

2. Second, each state's population is divided by the average district size, just as you did in the last activity. The whole number part of the quotient is used to assign seats.
   For example, if the average district size is 570,000, California would automatically be assigned 52 seats because

$$\frac{\text{Population of California}}{\text{Average district size}} = 29{,}839{,}250/570{,}000 = 52.34956\ldots$$

Similarly, Michigan would automatically be assigned 16 seats because

$$\frac{\text{Population of Michigan}}{\text{Average district size}} = 9{,}328{,}784/570{,}000 = 16.36628\ldots$$

and New Mexico would automatically be assigned 2 seats because

$$\frac{\text{Population of New Mexico}}{\text{Average district size}} = 1{,}521{,}779/570{,}000 = 2.66978\ldots$$

Also, Wyoming would have only the first assigned seat since the quotient of its population by 570,000 is a number less than one.

$$\frac{455{,}975}{570{,}000} = .79995\ldots$$

3. If there are seats left over after this is done for every state, the remaining seats are assigned to states *in order of largest fractional remainder*. Thus, New Mexico, Michigan, and California would receive additional seats in that order because the fractional remainders are ranked in order of size.

   .66978, .36628, .34956

   If there were two extra seats and only these three states in the country, New Mexico and Michigan would receive an extra seat but California would not.

   Note: If Wyoming is also included, it would not be ranked, since its one representative is already guaranteed in step one.

Now You Try It

1. Determine the distribution of 67 seats for the N-States by the Hamilton-Vinton method.

2. Determine the distribution of 59 seats for the M-States by the Hamilton-Vinton method.

3. Determine the distribution of the 84 seats for California, Florida, Nevada, Alaska, and Arizona by the Hamilton-Vinton method.

   You notice that, most of the time, the Hamilton-Vinton method gives the same answer as the Webster method. *But it doesn't always*. Try the following exercise.

Peculiarities of the Hamilton-Vinton Method

4. Using the column "Percent change 1980–90" in Table B, determine what the population in each of the five states California, Florida, Nevada, Alaska, and Arizona will be in 2010, assuming that each population increases by the same percentage in every ten-year interval after 1990 as it did in the ten years from 1980 to 1990.

5. Use these new population figures and the *Webster* method to assign 84 seats to these five states, assuming they make up their own country.

6. Use these new figures and the *Hamilton-Vinton* method to assign 84 seats to these five states, assuming they make up their own country.

Now assume that between 1990 and 2010 another area of the country grows fast enough so that these fives states together lose one seat in the House and are assigned only 83 seats.

7. Perform the same computation as in 5, using 83 seats.

8. Perform the same computation as in 6, using 83 seats.

You should notice that in going from 84 seats to 83 seats, one of the states *lost* a representative, even though it had a greater rate of growth than a state that did not lose a representative. But this happened only for the Hamilton-Vinton method and not for the Webster method!

9. Which state lost a representative using the Hamilton-Vinton method?

This is one of the reasons the Hamilton-Vinton method was not adopted by Congress in 1981.

10. Try the computations again for the same five states, but now assign 90 seats in the House. What do each of the two methods give?

The lesson is that no mathematical procedure can be considered correct, but one mathematical procedure may have a more desirable outcome than another. In any case, the argument goes on.

Student Handout 2-6

The Effects of Undercounting

It has been estimated that the 1990 census undercounted the total U.S. population by some 5,300,000 people. What's a measly 5,300,000, you might ask? After all, this many people represent only 2.1% of the population.

This exercise demonstrates the effects of an undercount on reapportionment.

1. Assume for the moment that the undercount was evenly distributed over the fifty states. Thus the actual population of each state is 2.1% higher than what the 1990 census figures say.

   a. Compute what the correct population would be for these six states: California, Pennsylvania, Illinois, Missouri, Wisconsin, and Arizona.

   b. Now pretend these six states make up their own country with a House of Representatives with 117 seats (the number they have in reality). Use the Webster method to recompute the distribution of the 117 representatives' seats.

   c. Did any state lose or gain a representative?

2. A California official has estimated that a full one million people were missed in California alone.[7] Had these people been counted, California's official population would be almost 3.4% higher than the census count. Assume that Arizona was undercounted by the same percentage and that the 1990 census figures were correct for the other four states.

   a. Calculate the adjusted populations for Arizona and California.

   b. Use the Webster method to apportion the 117 representatives, assuming the new population figures.

   c. Which states gained or lost representatives?

State your conclusions from this exercise.

---

[7] Felicity Barringer, "Administration Rejects Census Change," *The New York Times*, Tuesday, July 16, 1991, p. A16.

The Census and Redistricting for the House of Representatives

Overview

One of the important uses for census data is creating districts for the U.S. House of Representatives. Our state governments design voting districts to determine who will vote for the representatives that the state is allotted and, therefore, which candidates will represent which voters. This is an excellent example of census data being used to make important decisions.

This exercise enables students to experiment with creating voting districts and helps them understand how districts can be created intentionally so as to favor certain political parties or ethnic groups. These exercises have more than one interesting "answer."

In the second activity students experiment with the concept of population density by studying a map of the voting districts in their state.

Mathematical Concepts

area, population density

Skills

making decisions based on data and intention, reasoning from a set of given rules and assumptions

Vocabulary

district, connected, population density, reciprocal

Materials

copies of Handout 2-7, copies of map of Billows, map of your state's voting districts, almanac. Also useful are Michael Kidron and Ronald Segal, *The New State of the World Atlas*, 4th edition (New York: Simon & Schuster, 1991), and annual charts from The Population Reference Bureau, 1875 Connecticut Avenue, N.W., Suite 520, Washington, D.C. 20009, (800) 877-9881.

Time Needed

two class periods

Activity 1: Redistricting in Billows

Use Student Handout 2-7

In the first activity students experiment with creating voting districts in the fictional state of Billows. We offer two versions of the Billows map. In one, the towns border each other; in the other, there is county land that is not incorporated into the towns. Use the map that suits your students' geographic sensibilities.

If you want the students to work individually, give them three maps each and create different boundaries on separate maps. An extra copy of the map for scratch work might also be helpful.

This activity can also be done in groups. Give a different scenario on the handout to each group, and tell the groups to create districts according to the appropriate scenario that you gave it. As a follow-up activity have each group present its map to the class and compare the different results.

One way to do this is to have each group put its map on a transparency in a particular color (different colors for different groups) and then superimpose the maps from different groups using an overhead projector.

For more drama, ask a group of three or five students to act as the Billows Supreme Court. Each of the redistricting groups should present its plan to the Supreme Court. Emphasize that each group needs to write out a coherent rationale for the plan that it proposes, including maps and computations. The Supreme Court should then write an analysis of the different plans and vote on the plan each member thinks is best. For even more drama, assign different political party affiliations (Democrat, Republican, or Independent) to each of the members of the Supreme Court. The court's analysis could then contain majority and minority opinions.

A useful exercise might be to first have groups work on scenarios 1 and 2 and present their results to the Supreme Court, and then have the whole class work on scenario 3 (bipartisan scenario).

However you set up the activity, make sure students give explanations (written or oral) for the decisions they make.

Activity 2: Districts in Your State and Population Density

Use maps (3) of congressional districts or your own

For this activity you will need to procure a map of the voting districts in your state, or some other state, if your state has only one district. Maps are included for the states of Massachusetts, California, and Oregon. These maps are from *The Congressional District Atlas for the 100th Congress*, put out by the U.S. Department of Commerce. This atlas, as well as atlases for earlier Congresses, are obtainable from any federal repository library. (Check your local public library, state library, or local university. The state library will know which libraries are federal repository libraries.) This atlas also contains more detailed maps than the ones we have included, which will be important if you are

interested in having your students do this activity. These detailed maps are especially useful for those states with very small or very intricate districts.

Have the students determine, using thin graph paper, the approximate area of each district. They can determine area by putting the graph paper on top of the map and counting the number of squares in each district. (Alternatively, you can put the district map on an overhead transparency, so that students can place the transparency over the graph paper.)

Students can compute the area of one square according to the scale on the map and then multiply the area per square by the number (possibly fractional) of squares in a given district. Students will also need to know the number of people per district (total population divided by number of districts).

Once they have done this, have them determine the *population density* of each district. Population density is the total population of an area divided by that area (measured in square miles, say). This gives the average number of people that live in one square mile in the state. The *reciprocal* of the population density gives the average area per person in that district.

Ask the students to present their computations in a table and then lead a discussion in which students compare the different districts.

- Are there large differences in area for different districts?
- Are there large differences in population density?
- What might be some reasons for the differences in population density?
- What guesses do they have for why the districts have the shapes they do?

Follow-up

Using an almanac or charts from the Population Reference Bureau, ask the students to compare the population densities of different countries. Which countries have especially high or especially low population densities? (Be sure to check Hong Kong, Bangladesh, Canada, and Australia.)

Student Handout 2-7

Welcome to the State of Billows

Billows has a population of 350,000 people and a mixed economy of agriculture, manufacturing, and computer industries. After the last census it was determined that Billows has one representative in Congress for every 35,000 people, so the state needs to be divided into ten voting districts. The state legislature is the body responsible for determining a plan for redistricting, but this plan must be approved by the Billows Supreme Court.

The rules for creating districts are:

1. Each district must have 35,000 people.

2. Each city or town with fewer than 35,000 people must fall entirely within one district.

3. The area that determines a district must be *connected*, which means that it can't be in two or more separate pieces.

In each of the following scenarios you are asked to come up with a plan for redistricting. Use the attached map, which has the population identified by political party membership. Assume for this activity that party affiliations determine the way people vote in elections. The letters indicate:

D = Democrat
I = Independent
R = Republican.

(Independents' votes vary.) City limits are marked by hatched areas, and each figure equals 5,000 people. Your job is to create districts according to the guidelines in the given scenario and then to present your plan to your class (or to the Supreme Court made up of your classmates).

Scenario 1

In this scenario create voting districts assuming the state legislature is controlled by the Democrats. Your group therefore represents the Democratic Party.

- What areas of the map are problematical for Democrats? How will you deal with these areas?

- After you draw the district lines, what kind of advantage do the Democrats now enjoy?

- If people usually vote according to their party affiliation, how many people would consistently feel out-voted or under-represented in your plan?

- In your plan will each party have representation in Congress? Why or why not?

When you have finished the activity, write a careful statement of your redistricting plan, including reasons for the choices you made and answers to the above questions.

Scenario 2

In this scenario create voting districts assuming the state legislature is controlled by the Republicans. Your group therefore represents the Republican Party.

- What areas of the map are problematical for Republicans? How will you deal with these areas?

- After you draw the district lines, what kind of advantage do the Republicans now enjoy?

- If people usually vote according to their party affiliation, how many people would consistently feel out-voted or under-represented in your plan?

- In your plan will each party have representation in Congress? Why or why not?

When you have finished the activity, write a careful statement of your redistricting plan, including reasons for the choices you made and answers to the above questions.

Scenario 3

In this scenario you represent a bipartisan group. Your task is to create a districting plan that is as fair as possible.

1. First, decide what you think "fair" means. Write a careful definition of fair.

2. Devise a strategy for finding a fair plan, and then determine district lines.

   - Is your final plan fair according to your definition? Why?
   - What computations did you perform to ensure fairness?
   - In your plan will each party have representation in Congress? Why or why not?

When you have finished the activity, write a careful statement of your redistricting plan, including reasons for the choices you made and answers to the above questions.

Billows 1

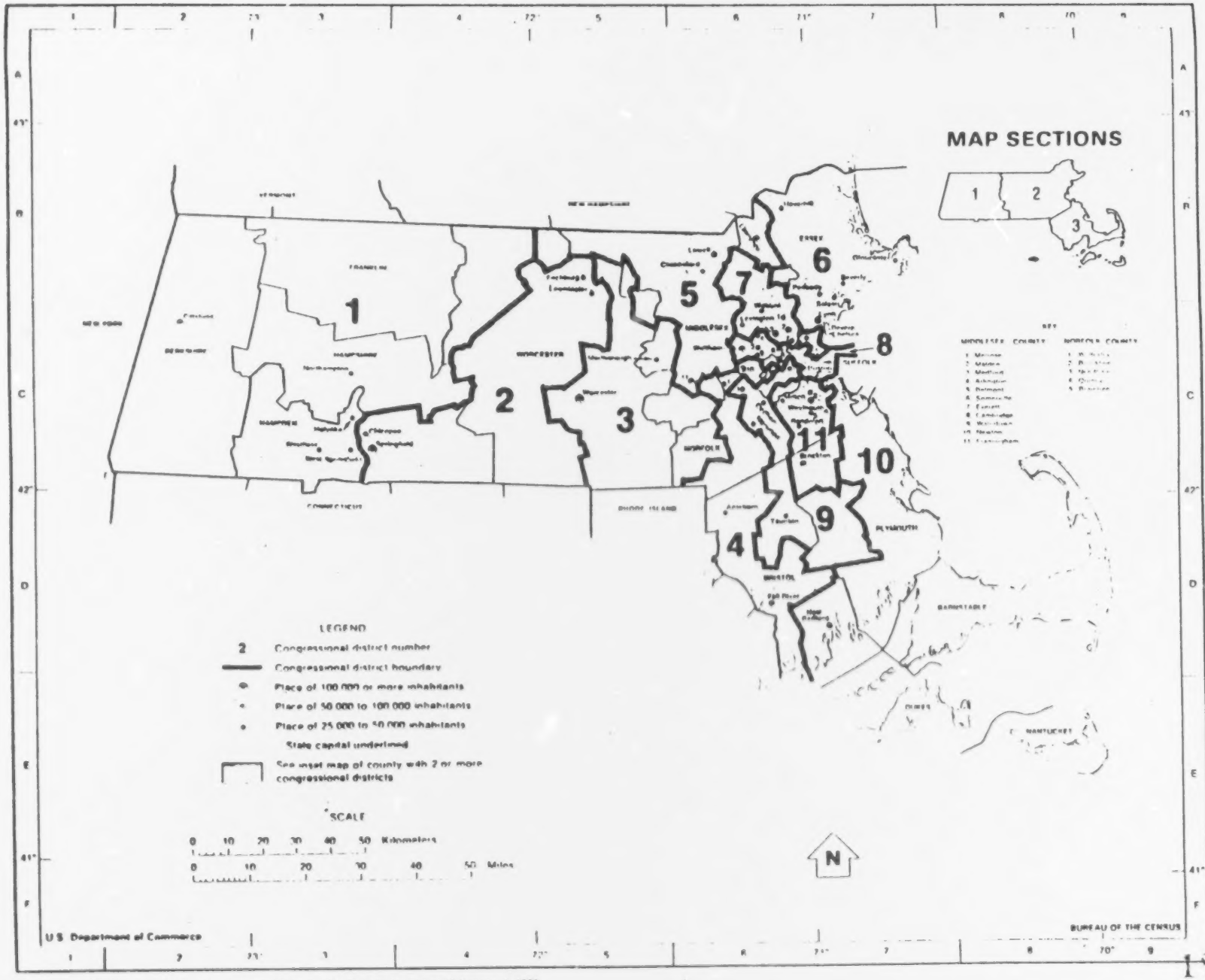
181

182

Billows 2

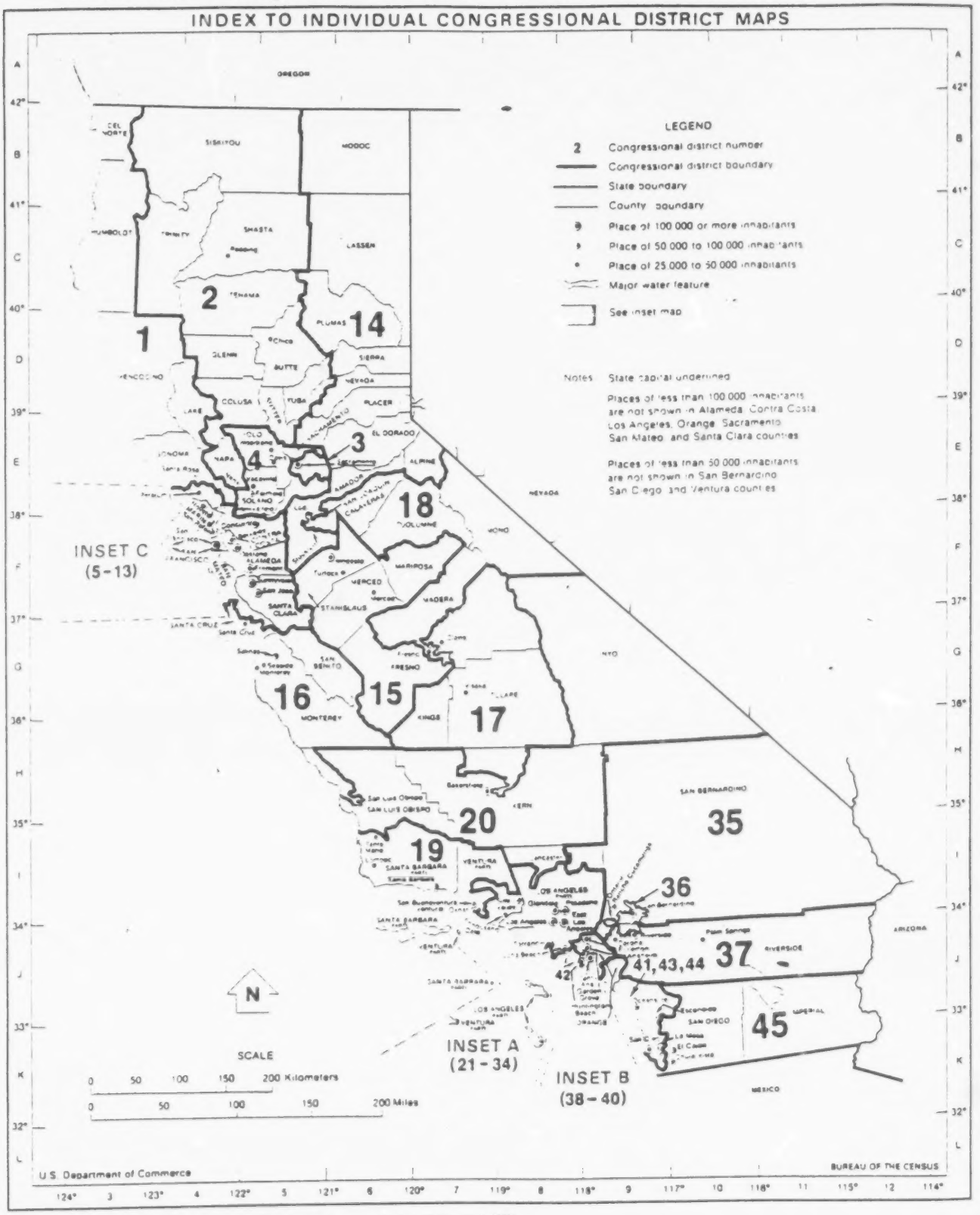
Tinseltown
Foxton
I-92
Kirkville
Bison
Formosa
Drew
Centerville
Growtown
I-17

MASSACHUSETTS — Congressional Districts, Counties, County Subdivisions (Towns), and Places — *(11 Districts)*



Congressional districts established December 3, 1981; all other boundaries are as of January 1, 1980

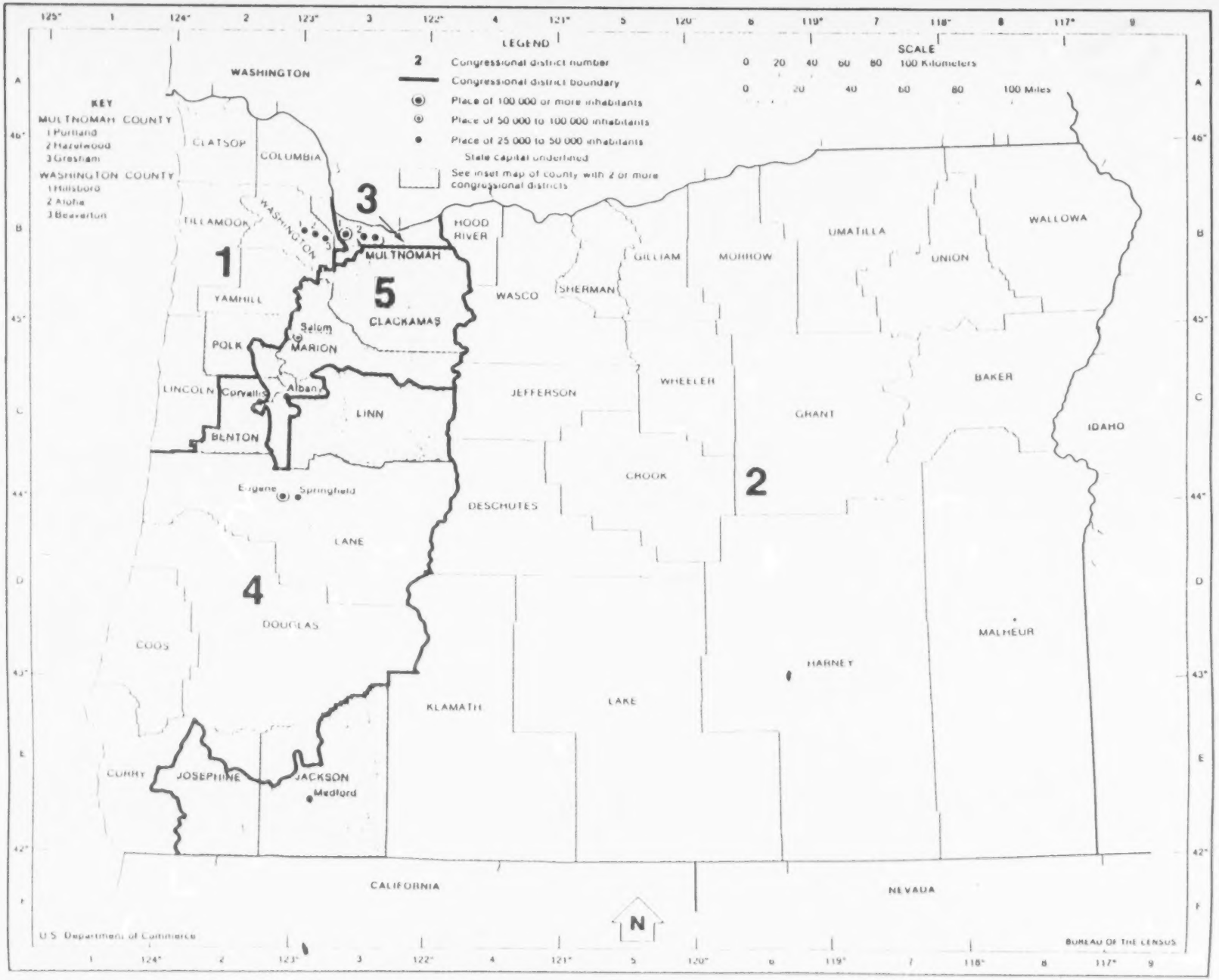
CALIFORNIA — Congressional Districts, Counties, and Selected Places — *(45 Districts)*


INDEX TO INDIVIDUAL CONGRESSIONAL DISTRICT MAPS
LEGEND
2 Congressional district number
Congressional district boundary
State boundary
County boundary
Place of 100,000 or more inhabitants
Place of 50,000 to 100,000 inhabitants
Place of 25,000 to 50,000 inhabitants
Major water feature
See inset map
Notes: State capital underlined.
Places of less than 100,000 inhabitants are not shown in Alameda, Contra Costa, Los Angeles, Orange, Sacramento, San Mateo, and Santa Clara counties.
Places of less than 50,000 inhabitants are not shown in San Bernardino, San Diego, and Ventura counties.
INSET C (5-13)
INSET A (21-34)
INSET B (38-40)
N
SCALE
0 50 100 150 200 Kilometers
0 50 100 150 200 Miles
U.S. Department of Commerce
BUREAU OF THE CENSUS

Congressional districts established January 2, 1983; all other boundaries are as of January 1, 1980.

157

OREGON — Congressional Districts, Counties, and Selected Places — *(5 Districts)*



Congressional districts established July 28 1981 all other boundaries are as of January 1 1980

Rates of Change

Overview

The purpose of this lesson is for students to learn about rates of change by observing, exploring, and computing trends and patterns in census data. This is one of the most important mathematical concepts we are introducing: rates of change are constantly used in mathematics and the sciences for understanding change over time. The lesson includes three discussions, a group activity on formula making, a graphing exercise, and a handout in which students are asked to use the techniques of the lesson to explore trends in actual census data.

Mathematical Concepts

rates of change, especially the difference between average increases or decreases and actual increases or decreases

Skills

reading and comparing tables of information, recognizing patterns, extrapolating, finding mean averages, developing and using a formula, using real numbers, graphing

Vocabulary

time interval, rate of change, rate of increase, decrease, negative increase, mean increase, average increase, average rate of change, formula, ordered pairs, difference, absolute value

Materials

graph paper, calculators, straightedges, Handouts 2-8, 2-9, 2-10, 2-11, and Table C

Time Needed

four–six class periods

To develop the idea of rate of change, lead a discussion in which students compare simple population statistics. The following three activities contain an outline for this discussion.

Activity 1: Opening Class Discussion: Comparing Changes in Population

Use Student Handout 2-8

This activity can be done either as a class discussion or a small group activity. If the students work in small groups, give them Handout 2-8. If you choose to do this as a class, or if you have a discussion following the group activity, the following suggestions may be helpful.

1. Write the following figures on the board and tell the students that the figures represent year and population of a city.

1980	260,000
1973	210,000
1970	170,000
1965	150,000
1960	100,000

Ask students what they notice about these figures. This can be done with the whole class or in small groups. Either way, have them record their responses.

Possible responses: The spacing between years and population figures is not always the same; population increases over time and increased more slowly between 1960 and 1970 than between 1970 and 1980.

2. If the students came up with only simple responses, encourage further responses by asking more complex questions.

- Did the population increase faster between 1960 and 1970 or between 1970 and 1980?

The question may seem easy to answer because the students have to compare only the amounts by which the population increased. However, it is possible that students may differ in their interpretation of the word "faster." If students interpret "faster" in the simplest way, as measured by change in population over time, then the population changed faster in the interval 1970–80 (90,000 in ten years) than in 1960–70 (only 70,000 in ten years). On the other hand, if students understand the question to be about *relative* population increase, then the population changed "faster" between 1960 and 1970, because the relative change in that interval, namely 70,000/100,000 = 70%, is greater than the relative population change in the period 1970–80, which is 90,000/170,000 = 53%.

The other two questions on Handout 2-8 may also be answered in different ways, depending on the interpretation of the question. In each case the answers are directly contradictory.

For simplicity, the notes in the following discussion assume the first interpretation of "faster," as population increases over time, not relative increase. This interpretation leads to the concept of slope and eventually to the concept of derivative in calculus. If students have differing interpretations and contradictory answers, however, this presents an excellent opportunity for them to learn that mathematics questions can sometimes have more than one answer. If students differ in their answers, have them explain their rationales very carefully, and highlight how the interpretations vary. It is important for you to listen for and encourage different ways of answering the questions.

This question will come up again in Activity 3 of "Doubling Time" (see Handout 2-13, questions 2 and 3), when students will find that the growth rate (population change over time) of the U.S. population has gradually increased since 1790, but that the relative growth rate has decreased.

- Did the population increase faster between 1970 and 1980 or between 1960 and 1965?

Ask the students to explain what they think this question means. Emphasize that there are different time intervals involved.

Giving an answer here is also not too difficult. The students are likely to come up with the following method for arriving at one: just double the amount by which the population increased from 1960 to 1965 (2 times 50,000) and compare that to the increase between 1970 and 1980 (70,000). Can they explain why they think this works?

Here is one explanation. If the population had increased at the same rate from 1965 to 1970 as from 1960 to 1965, then it would have increased by 50,000 in each five-year interval 1960–65 and 1965–70. Thus the population would have increased by 100,000 in the ten-year interval 1960–1970, whereas it increased by 90,000 in the decade 1970–80. Thus the population increased at a faster *rate* in 1960–65 than in 1970–80. *Note:* This method gives a hypothetical figure for the population in 1970 that is not accurate. This is okay since we are only comparing rates.

3. *The next step is crucial.* Ask whether the population increased faster during 1960–65 or 1970–73. (Notice that different time intervals are involved.) Does the method explained in 2 above work here? (No.)

   The easiest way to answer this question is as follows. The actual increase for 1960–65 is 50,000 and for 1970–73 is 40,000. How can we compare figures for different time intervals? *Answer:* Between 1960 and 1965 the increase is 10,000 per year, and between 1970 and 1973 the increase is 40,000/3 = 13,333.3 per year. Now the time interval is the same—one year. Since 13,333 is greater than 10,000, the population increased faster during the period 1970–73.

   Note: This method also involves hypothetical figures, namely, the figures 10,000 and 13,333.3. The population may not have actually increased by these amounts each year. These figures are, however, still useful for purposes of

comparison. (You could also make this clear by inventing figures for the years between 1960 and 1965, for example. See activity 2.)

4. Have students compare several other time intervals in the table, say, 1973–80, 1965–70, 1965–80, etc., using the same techniques.

Activity 2: Class Discussion: Finding Average Rates of Change

Use Student Handout 2-9

Here is a second way of looking at average rates of change. As with the last activity, you may want to conduct a class discussion or have the students work in small groups with Handout 2-9.

1. To take the discussion further, ask the students how the population changed between 1960 and 1965. One response (a likely one) might be: it increased by 10,000 people each year between 1960 and 1965. If the students respond this way, ask them for other possible scenarios.

2. Give the class the following scenario. (This may also be used as a homework assignment.) Suppose the population grew between 1960 and 1965 according to the following figures:

Year	Population	Increase over previous year
1965	150,000	
1964	120,000	
1963	130,000	
1962	90,000	
1961	110,000	
1960	100,000	

Have the students fill in the third column for each year beginning with 1961.

- What do they notice? (The increases change from year to year; they are sometimes positive and sometimes negative. Emphasize that a *decrease* isalso a *negative increase.*)

3. Next ask how they would determine the *average yearly increase.*

Answer: They would add up the individual increases (positive and negative) and divide by the number of steps (five). This number could be called the *mean* yearly increase. Emphasize that this is exactly the same type of average they computed in Part 1. There are five steps, and the sum of the changes is (from the bottom)

$$10{,}000 + (-20{,}000) + 40{,}000 + (-10{,}000) + 30{,}000 = 50{,}000.$$

Thus the *total change* from 1960 to 1965 is 50,000, and the average change is 50,000/5 = 10,000 people per year. This explains why the figure 10,000 is called the *average rate of change* of the population from 1960 to 1965. The rate changes from year to year, and 10,000 is the average change.

4. Ask students to find average rates of increase for other time intervals (1960–63, 1960–64, etc.) in the same way, for practice.

Activity 3: Algebra Exercise: Recognizing Patterns

Use Student Handout 2-9

This discussion completes the previous two discussions and gives an answer to question 5 of Handout 2-9.

1. In finding the average (or mean) rate of change in activity 2 above, we first computed yearly changes and then added all the changes and divided by 5. Ask the class: Is there a way to find the average rate of change (10,000 people per year), without doing all this work? You can also ask this question in the following way:

   - Is there a way to find the average rate of change without using any addition? (They may remember the answer given to the question in part 3 of activity 1.)

2. If they do not come up with it themselves, remind the class that the average rate of change in the last discussion can be computed in a simpler way:

   150,000 (pop. in 1965) - 100,000 (pop. in 1960) = 50,000.

   This gives 50,000, and dividing by 5 gives 10,000, the same answer we got in activity 2. Notice that we did not use any addition, only subtraction and division.

   - Now ask: Why does this give the same answer as the method we used in activity 2, adding the individual yearly changes in population?

3. To have the students come to an answer, write out each of the yearly changes as a difference:

Year	Pop. change	= difference
1965	30,000	= 150,000–120,000
1964	-10,000	= 120,000–130,000
1963	40,000	= 130,000–90,000
1962	-20,000	= 90,000–110,000
1961	10,000	= 110,000–100,000
Sum of yearly increases =	50,000	= 150,000–100,000.

- What patterns do they notice in this table?

You may want to have the students generate written responses to this question in small groups.

4. Once students have generated responses, try to have them see the following pattern. On the left side of the equal sign, the signed changes add up to 50,000 (the total change), as before. On the right the middle terms cancel: -120,000 cancels 120,000, -130,000 cancels 130,000, etc. In the end, what is left is 150,000 minus 100,000.

   This shows how the yearly changes add up to give the total change.

Activity 4: Formula Making

Use Student Handout 2-10

Now we are ready to find a general formula for average rate of change. Divide the students into small groups of three to four. The following instructions refer to the population table on Handout 2-10.

1. First ask the students to explain in their own words the short method for finding average rates of change.

2. Ask them to create a formula for the average rate of change of the population in a given time interval. Their formula should involve several variables. Emphasize that they will have to name the variables that go into the formula and will have to explain what each variable means.

3. Have them use their formula by computing the average rate of change of the population for each of the short intervals (that is, the intervals represented by consecutive entries) in the following table:

Year	Population
1990	260,000
1987	190,000
1983	210,000
1982	180,000
1980	170,000
1978	155,000
1972	90,000
1970	100,000

4. Tell the students to hand in a one- or two-page description of their answers to questions 1–3. These should contain a verbal description of the process for finding average rate of change, the answers to the computations asked for in 2, and the formula they came up with. They should explain the meaning of all the variables in writing and also illustrate the formula using the examples in 3 or using other examples that they invent themselves.

If you want your students to graph the information they have computed, see Activity 5.

Summary

The easiest way to find the *average rate of change* of a population over a period of time is to find the amount by which the population changed in that time and divide by the time interval involved, in the appropriate units. In symbols, if p1 and p2 are the sizes of the population at times t1 and t2,

average rate of change = (p1 - p2)/(t1 - t2).

In the above example we measured time in years, so every rate of change is expressed in terms of people per year. If we had measured in months it would be people per month.

Going Further

After the students have completed Activity 4, ask how they would compute the average *daily* rate of change of the population or the average *minute-by-minute* change. Would these be useful pieces of information? Why or why not?

Activity 5: Graphing

Use Student Handout 2-10

Rates of change can be shown concretely by graphing. This exercise has students graph the data from Activity 4 using line graphs. In this case line graphs are more appropriate than bar graphs, since they include pictorial representations of rates of change (slopes).

1. First review with students the method of graphing by coordinates.

   Explain how to set up the axes of a graph with the bottom line representing time and the vertical line representing population. Choose appropriate scales for the axes of the graph: one possibility is to say the horizontal axis is measured in single years from 1970 to 1990, and the vertical axis is measured in increments of 10,000, from ten 10,000s to twenty-six 10,000s. (You may want to consult other references for additional ideas about or more practice with graphing. There are also good computer software programs available: *Mecc Graph and Top Graph.*)

2. Give students graph paper and have them plot the points corresponding to the pairs in the table in Activity 4 (Handout 2-10).

3. Now tell the students to connect consecutive points with straight lines. This graph is called a line graph (or linear graph) of the points in the table.

   - What do they notice?

   In order to get good results for this activity, the students need to plot their points accurately and use a straightedge to connect them. Emphasize the care that it takes to get an accurate representation of the information presented in the table.

4. Tell the students to correlate the graphs they have just computed with the rates of change they computed in Activity 4 (third column in Handout 2-10).

   For example, consecutive pairs of points are connected by straight lines. Consecutive pairs are also used to compute average rates of change. Have the students label each straight-line segment with the rate of change that goes with those two points. So the segment joining (1970, 100,000) with (1972, 90,000) would be labeled with -5,000, which is the average rate of change of the population for the time interval from 1970 to 1972.

   - What do the students notice about this labeled graph?

   The most obvious response probably is that when the rate of change is positive the line slants up (from left to right); when the rate of change is negative the line slants down. Another thing students might notice is that the line is *steeper* (from left to right) when the absolute value of the rate of change is *higher*.

These observations are the starting point for the branch of mathematics known as *differential calculus.*

Activity 6: Rates and Trends over Time

Use Student Handout 2-11

The Student Handout 2-11 gives students the opportunity to apply the skills they have learned in this and previous lessons to understanding Table C: "Years of School Completed by Race and Sex: 1940 to 1980," which is part of the handout. This is adapted from the three pages of Table 83 in the 1980 U.S. Census Bureau publication PC80-1-C1, *General Social and Economic Characteristics.* In the table each column lists how many people completed exactly the number of years of schooling listed in the column heading, except for the first column and the last two columns which offer sumary data.

It is important to emphasize to the students that they need to read the headings of the chart and figure out what the entries mean by focusing on some specific examples. These are provided in exercise 1 on the handout, though you may also want to have them practice with additional examples.

Suggestions for Handling Questions

Help students understand that the "Percent High School Graduates" column lists the percentage of people on each line who have completed high school, including those who have completed more than high school. It is not the percentage who have completed *only* high school. They can calculate the number of people over twenty-five in 1980 who had completed high school in at least two ways. They can add the numbers in the "High School - 4 years" column and the three college columns, or they can multiply the percentage of high school graduates by the total population. If they do the latter, the result will be off slightly, since 66.5% was rounded from 66.47%.

In exercise 2 students are asked to study trends in years of school completed by gender and by race. Have students work in pairs for this activity. Students are given four problems, each with two sets of data. Be sure students understand that in each of the four problems their graph should have two lines or two sets of bars. Once they have completed the graphing exercise, have a couple of groups show the class their graphs for problem 1, another couple of groups for problem 2, etc. Discuss each problem further if any group's graphs are incorrect. Also discuss differences among the charts that are simply a matter of preference, such as different scales. With each set of data, ask students what trends they notice.

Students will remain in pairs for exercise 3. Using calculators, they will compute the rates of change for each of the data sets in problems 1, 3, and 4. Since each of these three charts has two lines, each with five points, there will be twenty-four computations. Have students divide the calculations, so each person has twelve to do. The handout includes two questions to use in a class discussion as a summary of the activity.

The last part of the activity, exercise 4, asks students to predict the census figures for 1990, 2000, and 2010 by race and/or gender. They can continue to use a set of data from

exercises 2 and 3, or they can select a new set of data to work with. If they select a new set, they will need to go through the steps in exercises 2 and 3 to predict the future trends.

It is up to the students to find an appropriate method for predicting trends. Methods they use are appropriate as long as they can support them. Be very clear with them that in making predictions they need to state their assumptions and carefully describe the method they used to come up with their figures. This point will need to be emphasized and (probably) restated.

As the final step in exercise 4, students should write a clear report that includes their computations, graphs, and predictions. You may also want to have them present their findings to the class.

An interesting follow-up discussion to the handout might center on the different methods the students came up with for predicting future trends.

(Note that on the chart the figures for white people added to those for black people do not equal the total population figures, except in 1940. For years after 1940 figures for other races are included only in the totals. Please see the footnotes on the chart.)

Going Further

Instead of having students focus on the column headed "Percent High School Graduates," you might have them compute their own column. For example, they could compute a column titled "Percent Completed Only through the 8th Grade." They would then have to add up figures in the appropriate columns of the table.

Student Handout 2-8

Rates of Change, Part 1

The following figures represent the year and population of a city. In answering each of the questions below, give as thorough an answer as you can. Explain clearly how you can justify your answer.

1980	260,000
1973	210,000
1970	170,000
1965	150,000
1960	100,000

1. Did the population increase faster between 1960 and 1970 or between 1970 and 1980?

2. Did the population increase faster between 1970 and 1980 or between 1960 and 1965? Why?

3. Did the population increase faster between 1960 and 1965 or between 1970 and 1973? Why?

Student Handout 2-9

Rates of Change, Part 2

In the same city, the population figures between 1960 and 1965 were:

Year	Population	Increase over previous year
1965	150,000	
1964	120,000	
1963	130,000	
1962	90,000	
1961	110,000	
1960	100,000	

1. Fill in the third column.

2. What was the average yearly increase in population between 1960 and 1965?

3. How could you compute the average yearly increase using only addition and division?

4. How could you compute the average yearly increase using only subtraction and division?

5. Why do you get the same answer in questions 3 and 4?

Student Handout 2-10

Finding a Formula for Rates of Change

1. State in words how to find the average rate of change of a population between two given dates.

2. Use your explanation in question 1 to write a formula for the average rate of change in a population between two given dates. List all the variables you need in the formula and what each variable means. Be prepared to explain how to use your formula.

3. Use your formula to compute the average rate of change in a city whose population is given below. Compute the rate of change for the intervals 1970–72, 1972–78, 1980–82, etc., up to 1987–90.

Year	Population	Average Rate of Change for interval ending in year
1990	260,000	
1987	190,000	
1983	210,000	
1982	180,000	
1980	170,000	
1978	155,000	
1972	90,000	
1970	100,000	

Student Handout 2-11

Rates and Trends over Time: Years of School Completed

Table C is adapted from Table 83: "Years of School Completed by Sex and Race: 1940–1980" of the 1980 census report and contains information about the number of years spent in school by individuals who were twenty-five or older at the time of the census.

Exercise 1: Understanding the Table

The first step in dealing with a new chart is to study the chart and make sure you understand what all the entries mean. For example, the figure 21285922 in the row 1970 and column, "High School, 1–3 years," represents the total number of people in 1970 who completed one to three years of high school and went no further with their formal education.

- What do the entries in the column headed "Elementary, 5 to 7 years" mean?
- What do the entries in the column headed "Percent High School Graduates" mean?
- How you would calculate the *total* number of people over twenty-five in 1980 who had completed their high school education? Note that the column headed "High School - 4 years" lists only those people who completed high school and went no further with their formal educations.
- What does "Median Years of School Completed" mean in the last column?

Exercise 2: Graphing the Information

Make the following graphs (each with two sets of data).

1. A line graph of the number of black people who have completed at least high school and the total population of black people twenty-five years and older, both from 1940–80.

2. A bar graph of the median years of school completed from 1940–80 for white people and the same for black people.

3. A line graph of the percent of high school graduates from 1940–80 who were male and the same for females.

4. A line graph of the total population from 1940–80 and the number of people who completed just zero to four years of school in the same period.

Exercise 3: Finding Rates of Change

Using a calculator, find the average yearly rate of change in each ten-year interval from 1940 to 1980 for the sets of data used in problems 1, 3, and 4 above.

- Do you notice any trends in the rates of change you have computed?

- Do you notice any correlation between the rates of change and the behaviors of the graphs?

Exercise 4: Predicting

If you had to predict what the 1990 or 2000 or 2010 census figures would be for any particular group, how would you do it?

Select a group of people that you could make predictions about using this chart: all people, all males, all females, black people, black males, black females, white people, white males, or white females (all age twenty-five or older).

As you devise a method for predicting new census figures, be careful to understand what assumptions you make. Write out a careful explanation for the method that you find. Use your method to predict census figures for 1990, 2000, and 2010. Write your explanation so that other members of your group can understand it. Include in your report your calculations, graphs, and predictions.

Table C: Years of School Completed by Race and Sex: 1940 to 1980

[Data for race groups in 1950 and all data for 1940 exclude Alaska and Hawaii. Data are estimates based on a sample; see introduction. For meaning of symbols, see introduction. For definitions of terms, see appendixes A and B]

United States Regions	Persons 25 years old and over	Elementary			High school		College			Percent high school graduates	Median years of school completed
		0 to 4 years	5 to 7 years	8 years	1 to 3 years	4 years	1 to 3 years	4 years	5 or more years		
TOTAL											
1980	132 835 687	4 755 239	8 837 241	10 665 203	28 277 314	45 947 035	20 796 975	11 420 499	10 137 981	66.5	12.5
Male	62 416 454	2 407 672	4 267 377	4 892 529	8 862 989	19 422 198	10 038 976	6 123 449	6 401 264	67.3	12.6
Female	70 419 233	2 347 567	4 569 864	5 772 674	11 414 525	26 524 837	10 755 999	5 297 050	3 736 717	65.8	12.4
BLACK[1]	13 195 318	1 083 778	1 550 293	933 110	2 876 194	3 861 896	1 785 985	584 710	519 352	51.2	12.0
Male	5 895 269	591 035	706 244	397 886	1 207 618	1 670 806	823 774	254 523	243 383	50.8	12.0
Female	7 300 049	492 743	844 049	535 224	1 668 576	2 191 090	962 211	330 187	275 969	51.5	12.0
WHITE	114 290 384	2 997 005	6 631 933	9 391 468	16 663 849	40 784 148	18 263 953	10 422 559	9 135 469	68.8	12.5
Male	53 941 163	1 507 020	3 257 197	4 339 723	7 315 405	17 174 006	8 831 698	5 660 839	5 855 275	69.6	12.6
Female	60 349 221	1 489 985	3 374 736	5 051 745	9 348 444	23 610 142	9 432 255	4 761 720	3 280 194	68.1	12.5
1970	109 899 359	6 039 314	11 032 712	14 015 364	21 285 922	34 158 051	11 650 730	6 657 604	5 059 662	52.3	12.1
Male	51 869 770	3 152 774	5 475 479	6 708 341	9 633 537	14 365 218	5 526 759	3 518 159	3 490 403	51.9	12.1
Female	58 029 589	2 887 40	5 557 233	7 307 323	11 652 385	19 792 833	6 123 971	3 139 445	1 569 259	52.8	12.1
BLACK	10 375 093	1 513 154	1 939 650	1 089 211	2 571 500	2 196 611	609 008	269 971	185 988	31.4	9.8
Male	4 713 703	833 731	902 602	478 444	1 078 331	941 409	283 422	103 756	92 008	30.1	9.4
Female	5 661 390	679 423	1 037 048	610 767	1 493 169	1 255 202	325 586	166 215	93 980	32.5	10.1
WHITE	98 245 635	4 376 311	8 953 261	12 816 399	18 517 629	31 617 691	10 903 616	6 395 963	4 786 765	54.5	12.1
Male	46 527 222	2 242 281	4 503 036	6 175 370	8 462 196	15 272 817	5 174 668	3 367 827	3 329 027	54.0	12.1
Female	51 718 413	2 134 030	4 450 225	6 639 029	10 055 433	16 344 874	5 728 948	2 928 136	1 457 738	55.0	12.1
1960	99 438 084	8 302 582	13 753 827	17 443 933	19 115 915	26 483 684	8 742 070	4 613 367	3 011 906	41.1	10.5
Male	47 930 513	4 506 179	7 019 007	8 516 320	8 973 036	10 154 325	4 122 689	2 510 097	2 128 863	39.5	10.3
Female	51 507 571	3 796 403	6 734 820	8 926 613	10 142 879	16 301 159	4 619 381	2 103 270	883 043	42.5	10.7
BLACK	9 053 945	2 153 765	2 193 536	1 166 103	1 722 251	1 168 863	371 377	277 871	(3)	20.1	8.3
Male	4 240 021	1 199 506	1 013 996	523 302	733 169	680 213	172 058	117 777	(3)	18.2	7.7
Female	4 813 924	954 259	1 179 540	642 801	989 082	488 629	199 319	160 094	(3)	21.8	8.4
WHITE	89 581 174	5 988 729	11 450 906	16 179 146	17 273 976	23 099 636	8 310 937	4 601 669	3 076 175	43.3	10.9
Male	43 258 756	3 212 461	5 945 544	7 941 872	8 177 224	9 587 939	3 918 369	2 422 493	2 053 157	41.6	10.7
Female	46 322 418	2 776 268	5 505 362	8 237 274	9 096 752	13 511 697	4 393 868	1 979 176	823 018	44.7	11.2
1950	87 483 480	9 445 870	13 910 700	17 706 275	14 800 555	17 663 545	6 258 775	5 284 580	(1)	34.3	9.3
Male	42 628 350	5 052 975	6 994 125	8 822 375	6 976 480	7 510 770	2 902 920	3 026 825	(1)	32.6	9.0
Female	44 855 130	4 392 895	6 916 575	8 883 900	7 824 075	10 152 775	3 355 955	2 257 755	(1)	36.0	9.6
BLACK	7 795 380	2 467 895	2 161 145	890 885	1 021 740	599 590	221 030	160 050	(1)	13.0	6.8
Male	3 729 785	1 334 925	988 940	398 795	432 905	251 610	98 130	69 170	(1)	11.7	6.4
Female	4 065 595	1 132 970	1 172 205	492 090	588 835	347 980	122 700	90 880	(1)	14.2	7.2
WHITE	79 338 785	6 884 250	11 692 065	16 769 940	13 738 320	17 004 570	6 019 935	5 107 960	(1)	36.4	9.8
Male	38 683 835	3 659 275	5 970 040	8 396 060	6 518 915	7 226 740	2 792 970	2 946 275	(1)	34.6	9.3
Female	40 654 950	3 224 975	5 722 025	8 373 880	7 219 405	9 777 830	3 226 965	2 161 685	(1)	38.2	10.0
1940	74 775 836	10 104 612	13 656 146	20 736 918	11 181 995	10 551 680	4 075 184	2 513 900	893 631	24.5	8.6
Male	37 463 087	5 550 390	7 007 812	10 631 478	5 332 803	4 507 244	1 823 981	1 374 647	646 581	22.7	8.6
Female	37 312 749	4 554 222	6 648 334	10 125 440	5 849 192	6 044 436	2 251 203	1 139 253	246 850	26.3	8.7
BLACK	6 776 313	2 782 498	1 993 780	790 158	578 403	296 367	136 503	71 520	16 026	7.7	5.8
Male	3 349 115	1 515 684	921 901	373 096	243 147	125 858	54 167	34 853	11 080	6.9	5.4
Female	3 427 198	1 266 814	1 071 879	417 060	335 256	170 489	72 336	36 667	4 946	8.4	6.2
WHITE	67 999 523	7 322 114	11 662 366	19 966 762	10 603 592	10 255 333	3 968 681	2 443 380	877 605	26.1	8.7
Male	34 113 972	4 034 706	6 085 911	10 258 382	5 089 656	4 381 286	1 769 814	1 339 794	635 501	24.2	8.7
Female	33 885 551	3 287 408	5 576 455	9 708 380	5 513 936	5 873 947	2 178 867	1 102 586	241 904	28.1	8.8

Totals include persons with years of school completed not reported. [2] Included in preceding category. [3] For 1940, includes Black and other races.

Adapted from Table 83 of the U.S. Census.

Doubling Time

Overview

The article "There's Nothing So Powerful as an Exponential Whose Time Has Come" by Donella Meadows gives some insight into the notion of exponential (or geometric) growth and how this mathematical concept can affect our decisions and our lives.

This lesson is divided into three sections. In the first activity students read the article and respond in discussion and in writing to questions about the article. The second activity is a class experiment on reaction time, to physically demonstrate the idea of exponential growth. This exercise may help students have a good "feel" for what exponential growth really means. The final activity includes a project on the doubling time of the U.S. Population.

Mathematical Concepts

doubling time (the length of time it will take a certain population to double at its current rate of growth), percentage growth, function

Skills

estimating, calculating percentage growth rates, doubling times, doing independent research, graphing, calculating rates of increase

Vocabulary

arithmetic progression, geometric progression, doubling time, exponential growth, rate of increase, growth rate, function

Materials

a copy of the article "There's Nothing So Powerful as an Exponential Whose Time Has Come" by Donella Meadows for each student; a copy of Handout 2-12 for each student; thirty-two nerf balls or other soft objects, such as sponges, that can be thrown safely in the classroom, and a container to put them in, such as a large basket or ordinary garbage can; Table D: U.S. population, 1790–1980

Time Needed

three class periods

Activity 1: Understanding Doubling Time

Use Student Handout 2-12 and "There's Nothing So Powerful as an Expeonential Whose Time Has Come"

Have students read the article "There's Nothing So Powerful as an Exponential Whose Time Has Come" and work on discussion questions either in groups or as homework. Either way, students can present their findings.

The students will need to know the concept of percentage growth rate, called *annual growth rate* in the handout for this activity.

The *percentage growth rate* is found by dividing the average rate of change of the population in a time interval (from t1 to t2, say) by the number of people in the original population (p1):

$$\text{percentage growth rate} = (\text{avg. rate of change})/(\text{original population})$$

$$= (\text{avg. rate of change})/ p1,$$

expressed as a percent.

Working with several examples will help to make this clear.

Taking the first example in Activity 1 of the rates of increase lesson, the average rate of change of our population from 1960 to 1965 was 10,000 people per year. In this example the percentage growth rate for the time interval 1960–65 was 10,000/100,000 = .1 = 10%. This means that every year (on average) the population grew by 10% of the original population.

The same interpretation applies to questions 4a and 4b of the handout. To bring out the difference between average rate of increase and percentage growth rate, you might ask the students for the actual increase in the number of radios sold from one year to the next, if the *annual growth rate* in sales is 2%. Of course they cannot answer this until they know the number of radios sold in the first year. Pose a hypothetical figure for them, say 300,000, and then have them compute that the increase in sales was 6,000. Going back and forth between actual increases (or average increases) and percentage growth rates will help them to see the difference clearly.

For question number 6 of the handout, you may want to have the students work in small groups and brainstorm the information they need and how to obtain it. (They will need to know the volume of the earth, how much oil would fill this volume, how much oil is currently being used.) *It is important not to give students the answers; let them find these things out for themselves.* This question provides an excellent opportunity for students to share ideas and do independent research.

Activity 2: Reaction Time to Exponential Change

Collect at least thirty-two soft objects, like nerf balls (or sponges). Have one person stand in the middle of the class; this person will attempt to catch nerf balls as they are thrown and then put them into a basket or garbage can. The other members of the class will throw nerf balls to the receiving student in the following way (and please be careful):

1. First have students throw a ball once every two seconds until all the balls are thrown. The person in the middle should retrieve each ball and put it into the basket.

   This is an example of arithmetic growth.

2. Start over. Have one person throw one ball, then two seconds later have one person throw two balls, then two seconds later have two people throw four balls, then two seconds later have four people throw eight balls, finally two seconds later have eight people throw sixteen balls. The person in the middle should attempt to retrieve all the balls and put them into the basket or can before the next balls are thrown.

   This is an example of exponential growth.

3. Repeat 2 with different people in the middle and different throwers.

Follow-up Discussion

- How did the reaction time of the receiving student compare in the two situations?
- How does this fit with Meadows's statement, "You don't get much reaction time when your problems grow exponentially"?
- Where does the term *exponential growth* come from?

Activity 3: Doubling Time of the U.S. Population

Use Student Handout 2-13 and Table D

In this activity the students compute and graph the average rate of increase of the U.S. population in every ten-year interval from 1790 to 1980, the percentage growth rates for the same intervals, and the doubling time as computed from each of these different growth rates. The point is for them to see how the growth rate has changed, along with the doubling time. (In general the annual percentage growth rate has decreased steadily and the doubling time has increased.) If done correctly, their graphs will show noticeable declines in the growth rates in decades in which the United States was at war or in the middle of the depression. This is one of the points they may notice in answering question 5.

Students may make the mistake of using the percentage growth rate for an entire decade to compute doubling time. If they do this remind them that the formula for doubling time requires using an *annual* growth rate. Once they compute the rate of change in the population over a ten-year interval and convert it to a percentage, they need to divide by ten to find the annual growth rate.

Perhaps the best answer to question 7 is that the growth rate that most nearly represents how the population is changing *now* is the rate for the interval 1970–80. This

answer most nearly reflects the current laws and practices regarding birth rates and death rates (for example, immigration laws, health practices, family size).

But other answers to question 7 are possible. It is possible to argue that only more long-range rates should be used since anything can happen in a short time interval. There is no way of knowing if the current rate will continue. Thus a long-range figure is safer.

Emphasize in the follow-up discussion to this activity that no answer is correct, since no one can really predict accurately how the population will change. In fact, people invest a lot of money in the attempt to generate good predictions.

Going Further

This activity would be excellent background for reading the book *Limits to Growth* (or using *The New State of the World Atlas;* see References). It would be interesting to compare the predictions in the book with the actual changes in U.S. (or world) population since the time the book was written. The same comparison could be made for other factors studied in this book, such as food production and consumption, energy resources, etc.

It would also be interesting for students to compare the history of the growth rate of the United States (given in the graph the students create; call it the *growth-rate curve*) with the growth-rate curve of developing nations.

Student Handout 2-12

Doubling Time

Read the article "There's Nothing So Powerful as an Exponential Whose Time Has Come" by Donella Meadows and consider the following questions.

1. Describe in your own words the author's main points about doubling.

2. According to the author's figures on the population of Mexico, how many years will it take for the population of Mexico to reach 672 million? How many times will the population have doubled when it reaches 672 million?

3. If you were living in Mexico, in what ways would this increase in population affect your life?

4. In the article doubling time is given by the approximate formula:

   doubling time (in years) = 70 ÷ annual growth rate (expressed as a percentage).

   Find the doubling time in the following situations:

   a. If the number of radios sold in the United States increases by 2% a year, how long will it be before the number of radios sold in a given year doubles?

b. If the number of sales of radios increases by .05% per year, how long will it be before the number of radios sold doubles?

c. What rate of increase will produce a doubling time of 50 years? 30 years?

5. Describe the relationship between the annual growth rate and the doubling time in words. (Try coming up with sentences like "When the annual growth rate goes up. . ." or "The more the annual growth rate goes up. . .")

6. In the paragraph that begins "Exponential growth makes the cupboard bare very fast," the author states that if the earth were filled with nothing but oil, it would all be gone in 342 years.

   • What information do you need to be able to check that 342 is the correct figure?

7. Answer the same question about the sixty-one years the author states it will take to use up all of our available coal.

There's Nothing So Powerful as an Exponential Whose Time Has Come

by Donella Meadows

The reason environmentalists are often so gloomy is that they know what the word "exponential" means.

"A lack of appreciation for what exponential increase really means leads society to be disastrously sluggish in acting on critical issues," said Dr. Thomas Lovejoy of the Smithsonian Institution in a speech that has been reverberating through the environmental community. "I am utterly convinced that most of the great environmental struggles will be either won or lost in the 1990s, and that by the next century it will be too late."

What's he talking about? What does exponential increase mean?

It means growing like this: 1, 2, 4, 8, 16, 32. Doubling and then doubling again and then doubling again. Everyone understands that, right?

Not really, not at a gut level. For example: suppose you agree to eat one peanut on the first day of the month, two peanuts on the second, four peanuts on the third, eight peanuts on the fourth, and keep doubling every day. How long do you think you can keep going? How long will a pound can of shelled peanuts last you. The first pound of peanuts will be gone on the ninth—you'll eat half the can that day and feel pretty queezy. On the tenth you'll eat a whole pound, if you can, which I doubt. By the fifteenth you'll be scheduled to eat 32 pounds of peanuts. You'll have to eat roughly your own weight in peanuts by the 17th; on the 21st the total will have risen to one ton; and by the end of the month, assuming a 30-day month, it will be 500 tons.

Just a few doublings add up ferociously fast—that's what Thomas Lovejoy was saying. Mexico, with a population of 84 million and a doubling time of 29 years, will, if it keeps that up, grow to 168 million in 29 years and to 672 million within the lifetime of a child born today.

Until the 1970s world oil consumption was growing at 7 percent per year. That means doubling every 10 years. (The doubling time of anything growing exponentially is 70 divided by its annual growth rate—70 divided by 7 percent is a 10-year doubling time.)

Exponential growth makes the cupboard bare very fast. Even if the entire earth were filled with nothing but high-grade crude oil, if we used it with an annual growth rate of 7 percent, it would be gone in 342 years. There's still plenty of oil around now, but we've been burning it faster than we've been discovering it for 20 years now.

You may have heard that we have 1000 years' worth of coal. If we burn 7 percent more of it each year than the year before (which we may well do, substituting it for the disappearing oil), it will last just 61 years, and it will bring on global climate change much faster than even the worst pessimists are now expecting.

Said Dr. Lovejoy, "I find to my personal horror that I have not been immune to naivete about exponential functions. While I have been aware that the. . . loss of biological diversity, tropical deforestation, forest dieback in the northern hemisphere, and climate change are growing exponentially, it is only this very year that I think I have truly internalized how rapid their accelerating threat really is."

You don't get much reaction time when your problems grow exponentially.

We are emitting carbon dioxide and several other greenhouse gases in the atmosphere exponentially. We are clearing tropical forest at an exponential rate. The human population is growing exponentially. Human energy use, human production of synthetic chemicals, deserts, and trash are growing exponentially. Our economy is growing exponentially, and we cheer it on, although an economic growth rate of, say, 3.5 percent per year means another whole industrial world plopped down on top of this one in just two decades.

We can't keep it up. If we understood the consequences of exponential growth, we wouldn't even want to try.

Reprinted by permission from Donella Meadows, *The Global Citizen*, (Washington,D.C.: Island Press, 1991).

Student Handout 2-13

Research Project: Find the Current Doubling Time of the U.S. Population

1. Use the Table D: "U.S. Population 1790-1980" to compute the rate of increase of the U.S. population for every ten-year period since 1790.

2. Express the rate of increase for each period as a percentage of the original population (the population of the United States in the first year of the given time interval). We'll call this the percentage growth rate, or just growth rate.

3. Graph both the actual rate of increase and the percentage growth rate as a function of year.

4. How has the rate of increase in the U.S. population changed since the first census was taken? How has the growth rate changed?

5. What are some factors that may have contributed to the changes in the rate of increase? in the growth rate?

6. Use the figures you have computed to calculate the doubling time of the U.S. population, based on the growth rate in the interval 1970–80. Repeat this computation for each ten-year period since 1790 and graph the results. How has the doubling time changed since 1790?

7. Which time interval do you think is the best to use in computing the *current* doubling time? Is 1970–80 or 1900–80, or even 1790–1980 the best time interval, in your opinion? Why? Compute each of these doubling times and compare.

8. Based on the most reasonable doubling time that you find, estimate what the population of the United States will be in 50 years; in 100 years.

Table D: U.S. Population, 1790–1980

Year	Population	Year	Population
1790	3,929,214	1890	62,947,714
1800	5,308,483	1900	75,994,575
1810	7,239,881	1910	91,972,266
1820	9,638,453	1920	105,710,620
1830	12,866,020	1930	122,775,046
1840	17,069,453	1940	131,669,275
1850	23,191,876	1950	151,325,798
1860	31,443,321	1960	179,323,175
1870	39,818,449	1970	203,302,031
1880	50,155,783	1980	226,545,805

From *Statistical Abstract of the United States 1990,* 110th edition (Washington, D.C.: U.S. Department of Commerce), p. 7.

Cross-Referencing Census Data

Overview

The following activities offer students the opportunity to interpret data, observe trends in the U.S. population, and predict future possibilities using more than just a single set of data. Often simple information is offered to people as citizens or consumers when more data would yield a deeper understanding. Students' abilities to evaluate data as well as their abilities to conduct research and investigate will be stronger with cross-referencing skills.

Mathematical Concepts

rates of change, ratios, percentages, proportions, linear equations

Skills

reading tables of information, applying information from one table to another, inferring additional information based on the combined data, interpreting, applying raw data to ratios to determine new raw data, working with percentages and rates of change, solving linear equations, using a calculator

Vocabulary

ratio, per, percentage, extrapolate, proportion

Materials

calculators, and each group of students needs one set of these three charts:

Table D: U.S. Population 1790–1980, *Statistical Abstract of the United States 1990,* 110th edition, U.S. Department of Commerce, p. 7
Table E: "Persons 16 Years and Over in the Labor Force by Sex: 1950–1980" (Numbers in Millions), *1980 Census of Population: General Social and Economic Characteristics,* p. 1-10m
Table F: "Sex Ratio: 1900–1980," *1980 Census of Population: General Population Characteristics,* p. 1-15

Time Needed

three-five class periods

Activity 1: Cross-Referencing

Use Tables D, E, F

Step 1

Have students work in small groups of three to four. Give each group one copy of each of the three tables. Students will be reading, applying, and interpreting the data. The tables can be considered in any order; there is no one correct sequence. You may let them experiment to find their own order or have each group work on the data in a different order.

Have students describe each table in simple sentences and record their sentences, grouped by chart. Ask students to write at least five sentences for each table. For example, for Table E students may say, "The number of men in the labor force went up from . . . in the time period from . . .," "The number of women in the labor force went up also, with a greater rate of increase." There are *many* such sentences.

If students need help starting you can offer them questions for each table, such as:

Suggested Questions for Table E

1. What general trends can you determine from the data presented in this graph?

2. What is the rate of increase for the male population in the labor force from the years 1950 to 1980? (0.62 million per year.)

3. What is the rate of increase for the female population in the labor force from the years 1950 to 1980? (0.94 million per year.)

4. What could be some of the reasons for these results?

Suggested Questions for Table F

1. What does the term *ratio* mean as it is used in this graph?

2. What does the number *102.6* in 1930 mean? What mathematical steps did the Census Bureau use to derive that number? (It took the number of males, divided it by the number of females, and multiplied by 100.) Why did it use the unit of 100? (Help students understand that in units of 1, let's say, that there was 1.062 males for every 1 female, we are left wondering what 0.062 of a male is. Units that are too small in this case lose their meaning.)

3. What has been the trend in the ratio of males to females from 1900 to 1980?

4. What are some of the causes that could lead to this trend?

5. If this trend continues, what might the percentages look like in 1990? 2000? 2010? Give specific computations to support your number.

Step 2

1. After students state what information is presented in each chart, encourage them to state what information is *not* presented. For example, from Table E, they will know that 16.4 million women were in the labor force in 1950. But they do not yet know how many women were *not* in the labor force because they do not yet know how many women were in the population. Maybe there were only 16.4 million women in the United States that year!

   Students should write at least five sentences or questions that clarify what they cannot determine from any one table. Once again, there are many possible responses. Have students record these on a separate list.

2. Next, encourage them to use statistics and conclusions from one table to help them extrapolate new information from the others. Perhaps they will be able to fill in some of the gaps they just discovered.

Some questions to begin with are:

- Can you write the numbers on the U.S. population chart in the form of the numbers on Table E? How would you do it? (226,545,805 population in 1980 would be 226.5 million). Can you write the numbers represented on Table E in the form of the numbers on the U.S. population chart? (Yes and no. Yes, because 61.4 million represents 61,400,000 in a rounded-off form. No, because we do not know what the actual number was. The actual number could have been 1,000 or 20,000 or even 49,999 higher or 50,000 lower.) When is rounding off useful and why?

- How many females were in the population in 1930? Do you have enough information in the tables to answer this question? (Yes, there is enough information, and the answer can be computed in at least two ways to answer the question.) Encourage students to experiment and come up with various ways to answer the question. Some students will develop more or less formal methods. It is important for them to see that there are many tools to choose from in mathematics, to have their own successful methods confirmed while being introduced to other methods, and to find methods that build on the skills with which they are familiar. Below are two different methods, both dealing with 1930 and using the population chart and Table F.

Method A: Using Percentages

If there were 102.6 males per 100 females, then females comprised 49.36% of the population.

$$100/202.6 = .493583415597$$

Round this off to .4936 or 49.36%. If the twelve-digit decimal is used, the calculation results in the same answer as in method B below. It does not make sense to use more than four digits in this case because the figures in the table (102.6 males per 100 females) were rounded off already.

122,775,046 (total population in 1930) • .4936 = 60,601,763 females in the United States in 1930

Method B: Using Linear Equations

Let f be the number of females, let m be the number of males. There are two numerical relationships between males and females in our data.

$m + f = 122,775,046$ (total population in 1930)

$m/f \cdot 100 = 102.6$ ratio

Solve for m in both equations.

$m = 122,775,046 - f$

$m = 1.026 \cdot f$

Set these two equations equal to each other:

$1.026 \cdot f = 122,775,046 - f$

$2.026 \cdot f = 122,775,046$

$f = 122,775,046/2.026$

$f = 60,599,726.55$

Round that off to 60,559,727 females in the population in 1930.

- Did the number of males decrease in the population from 1930 to 1940? Is there enough information in the tables to answer this question? (There is enough information in the population chart and Table F. The number of males in the population did not decrease; it increased from 62,215,319 to 66,096,927. The ratio decreased, but the actual number increased.) Have students explain how the ratio can decrease while the actual number increases.

- How many females were *not* in the labor force in 1980? How many females aged sixteen or over were not in the labor force in 1980? Is there enough information in the tables to answer these questions? (Assuming that the Census Bureau defines

being in the labor force as being in the labor force at or over age sixteen, then the first of these questions can be answered; the second cannot be answered from the information given.) What other information is necessary to answer the second question? (We would need to know the number or percentage of women in the population under age sixteen or age sixteen and over.)

Step 3

Ask the groups to present their findings to the class. This offers them the opportunity to communicate mathematical ideas, using relevant vocabulary. There are many different stories that students will be able to tell depending on which table they started with and what details they focused on. Encourage them to give a reasonable amount of detail so students in other groups will fully understand several different ways of combining data and extrapolating new information. They can demonstrate the methods they used, present their own charts, or state their conclusions.

Activity 2: Research Interviews

As a homework assignment, have students interview parents or other people and/or do background reading to gain information about why some of the changes described by the data took place. What world events or other developments could have affected the population figures? They might discuss world wars, medical advances, immigration, influenza epidemics, or many other possible influences. The interviews and research will help students understand the context of the data with which they have been experimenting and see that these numbers represent real people.

This activity can be organized in several ways. Each group of students can focus on a specific decade, for example. Or each group could be responsible for finding out about a particular field, such as those listed above (wars, medical advances, etc.). After the initial interviews and research, form new groups that include one student from each of the former groups. Using this "jigsaw" technique, students can pool their knowledge to gain a larger perspective on the question "What might be some of the causes for the trends seen in the graphs?"

Have students write questions before they conduct interviews or do research to help them focus their inquiry. After pooling their information in their temporary jigsaw groups, they might have new questions to look into as well.

The results of the interviews and research can be presented through a class discussion, through written reports, or both.

Activity 3: Extrapolating New Data

Ask the students to graph the number of females and males in the total population from 1900 to 1980. They will need to use Table D and Table F. They will be creating an entirely new chart based on data they have extrapolated from the other two.

Table E

Persons 16 Years and Over in the Labor Force by Sex: 1950–1980 (in millions)

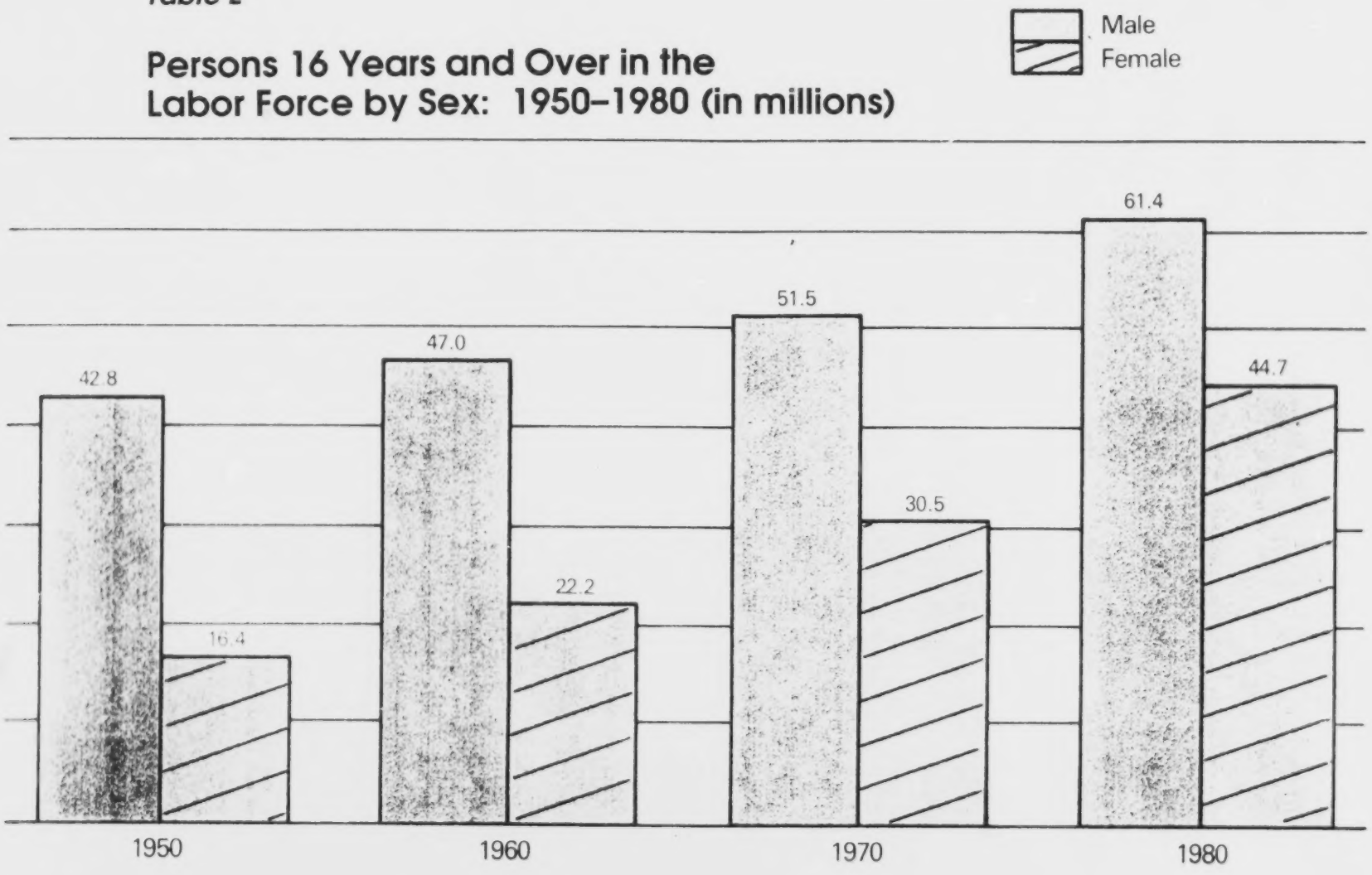
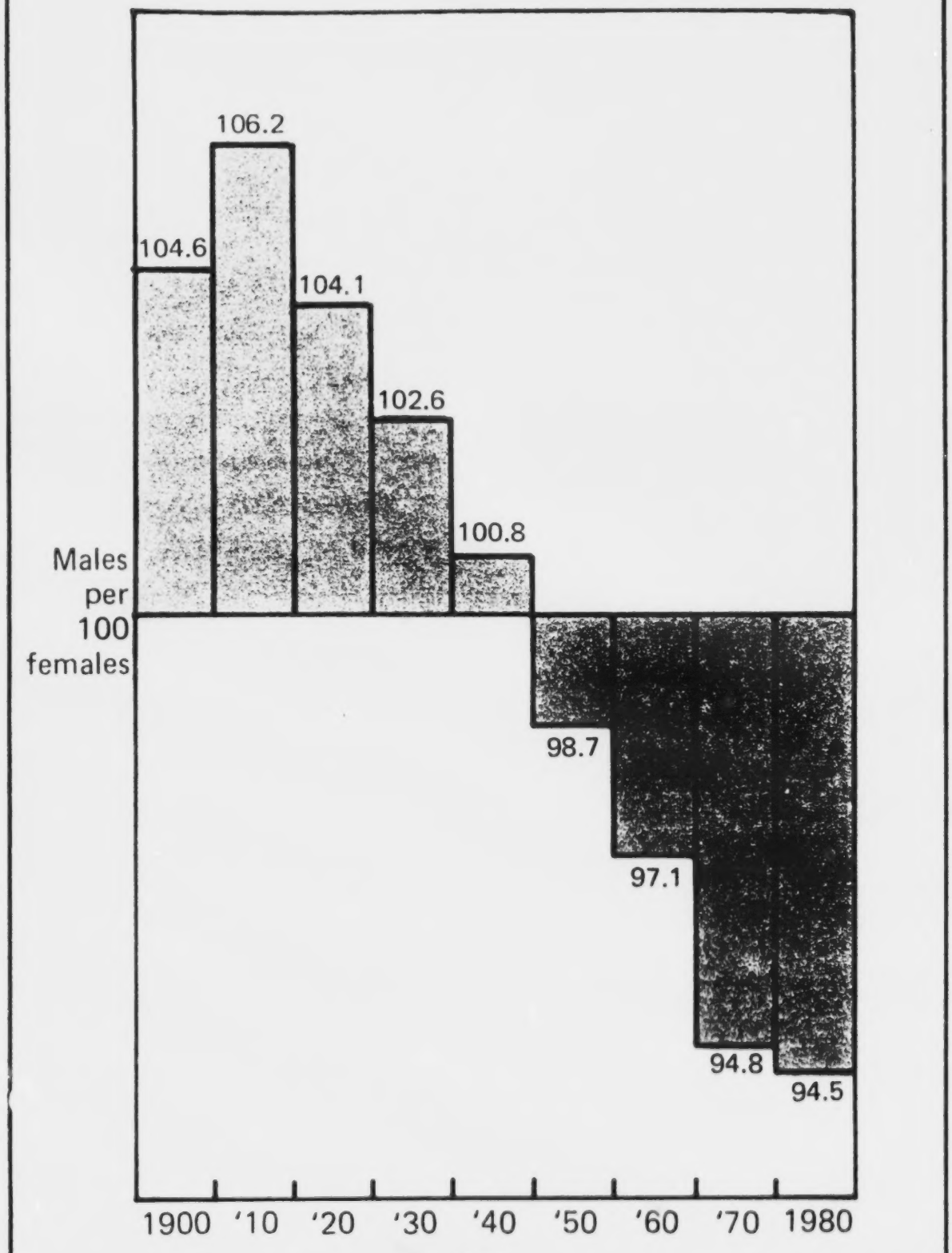


Table F

Sex Ratio: 1900 to 1980



Creating a Public Rail Transportation System for Los Angeles County

Introduction

This culminating lesson contains two activities and two more complex options for further investigation. The goal is to use information from the U.S. Census bureau volume *Journey to Work* to design a public rail transportation system for Los Angeles County. The lesson consists of:

1. a starter activity, in which students place yarn or draw lines from city to city (within Los Angeles County) to indicate numbers of people traveling from residential locations to places of work

2. a preliminary design activity, in which students design a public rail transportation system for Los Angeles County, based only on numbers of people traveling between home and work

3. Option 1 for further investigation, in which students modify their preliminary design by determining the costs of construction and experiment with operating costs and revenues from fares

4. Option 2 for further investigation, in which students modify their preliminary design by working with actual distances, rates, and time schedules

In Option 1 the students have to design the system to operate within a budget, while in Option 2 they have to move specific numbers of people to their destinations by specific times. In each case students should be encouraged to start with their preliminary design and modify it as they go. Options 1 and 2 both depend on the two preliminary activities but are independent of each other.

Student groups can also work on different options simultaneously. It might be intriguing for students to see that focusing on different themes can result in different subway systems.

Whether students stop this project after the preliminary design, after Option 1, or after Option 2, have them make final presentations. As described in Handout 2-14, have the students who are not currently presenting pretend that they are the Los Angeles County Transportation Board. Students who are presenting their final proposals should be encouraged to present diagrams, budgets, timetables, maps, and any other evidence supporting their plan.

Activity 1: Mapping it Out

Overview: In this lesson students, working in small groups, match the data represented in the *Journey to Work* census tables to a map of Los Angeles County.

Skills: reading a census table, comparing data, reading and locating points on a map, recognizing patterns, making concrete models of numerical information

Materials: (for each group) a Los Angeles County map, Table H adapted from the census volume *Journey to Work: Characteristics of Workers in Metropolitan Areas*, and either yarn and tacks (to be used on bulletin boards) or yarn and tape (to be used at desks) or eraseable colored pencils (see instructions below)

Time Needed: three–five class periods

1. Separate the class into groups of three. Give each group a copy of Table H and a Los Angeles County map. Additional materials will depend on your classroom design. Following are some options for applying data from Table H to the map:

   a. yarn or coat thread and clear tape

   b. yarn and tacks, if bulletin boards are available

   c. erasable colored pencils

   d. coat thread and an adhesive stick (glue like post-its)

   Students can use different colors of yarn to represent different numbers of travelers, or they can use one color only of coat thread to represent a fixed number of travelers.

   The point of this task is for students to create a visual representation of the data in Table H. Note that Table H is a condensed form of Table G. With this 9 x 9 grid, students will need to consider 81 travel routes. They may decide that some are insignificant and need not be represented on the map. You might pose the question "How many travel routes would we need to consider if we used Table G instead of Table H?"

   Using different materials will obviously produce a different kind of representation, so you need to choose an option that will be effective with your students. If students use yarn or thread of different colors, suggest that they use darker colors for higher numbers, so that the high-traffic corridors stand out. If they use thread of one color, have them assign a consistent value to a strand of thread. In this case, high traffic routes would be represented by multiple strands.

2. Now pose the following problem to your students. Los Angeles County is developing a modern public transit system. It is the students' job to design a county-wide trolley or subway system.

   The first step is to study Table H. They should first make sure they understand what the entries mean and get a feeling for the numbers by locating some of the cities listed in the table on a map of Los Angeles.

   The next step is to start representing the data in Table H on the map using whatever method you have selected for them. The task is to match where people reside with where they work on the map of Los Angeles County. The students should decide on a system for representing travelers and then mark a path from the residential location A to the city or place of work B, to indicate the approximate number of people who live in A and work in B. For example, a piece of red yarn placed between Alhambra City and Carson City might mean that about one hundred people travel from Alhambra City to Carson City for work and back again.

   As they begin marking paths between appropriate spots on the map, encourage them to be careful and consistent. They may find that they need to reassign yarn values once they have worked with the data.

   Encourage them to organize the data in any way that makes the activity more efficient or more manageable.

3. To do the task students may delegate subtasks within the group, write up a chart of their decisions about yarn colors, or organize a system for recording which data they have dealt with. Give students enough time to experiment, fumble, and develop their own systems. Offer suggestions to groups when they are really struggling.

Discussion Questions

(for whole class, small group discussion, or individually written responses)

- Do you see any patterns in the direction people travel to work?
- Where do most people go to work?
- Can you differentiate areas of your map that are primarily residential and primarily commercial?
- How did your group get the task done? Did you divide roles? Did you invent a recording system?

Activity 2: Developing a Preliminary Plan

Use Student Handout 2-14 and Table H

Overview: Based on Activity 1, students create a transportation map and present their design to the class. This activity is designed to give students an opportunity to interpret data, assign it meaning, and apply this information to decision making.

Skills: decision making based on numerical data, recognizing and applying patterns, presenting data and logical arguments to support decisions

Materials: (for each group) Los Angeles County map and tables from the Activity 1, Handout 2-14. Make available some of the following: oaktag, large sheets of newsprint, magic markers, meter sticks, pencils, blank overhead transparencies

Time Needed: two–three class periods

In this activity students design a transit system map of lines and stations only. They will deal with numbers of trains, cars, and riders (as well as costs) later. Now have them focus on their *dream* transit system.

Emphasize the following points to ensure careful planning by each group of students.

1. Students need to carefully examine the maps and tables that they worked with in Activity 1 before creating any design.

2. The next step entails deciding where to place different trolley and subway transportation lines and stations. Students should record all their reasons for making their decisions. These reasons need to follow some logical order. This is important for supporting their plans in oral presentations.

3. When each group has developed a plan, have students present their designs to their classmates, who can play the role of the Los Angeles County Transportation Board. In the presentation each group will explain its plans and the rationales for its designs. Again, careful consideration needs to be given to the logic of the designs. A presentation might include diagrams of the transportation lines and stations as well as tables of figures for supporting the rationale. Emphasize that the details are important for a clear understanding of the groups' methods and motives.

Discussion Questions

Conduct a follow-up discussion after the presentations. During the discussion emphasize that different people often give different interpretations of the same set of data. Encourage students to compare their rationales and decisions.

- What were the top priorities for each group?
- Did any of the groups consider some rationales to be more important than other groups considered them to be? Why? What effects did that have on their plans?
- Did any of the groups come up with similar design plans? Were those designs based on similar or different rationales?
- What mathematical techniques did students use? Were any of the rationales based on mathematical arguments?

Table G: Journey to Work Data for Los Angeles County

Top left quarter

To find the number of people going from their home in city X to work in city Y, look for city X in the first column and city Y in the first row.

Work / Home	Alhambra	Baldwin	Bellflower	Burbank	Carson	Cerritos	Compton	Downey	El Monte	Glendale	Hawthorne	Inglewood	Lakewood
Alhambra	5,177	33	10	143	116	102	98	100	631	262	30	80	14
Baldwin	175	2,483	13	156	23	36	40	71	1,267	61	25	15	11
Bellflower	10	9	3,324	25	623	903	677	1,928	100	13	130	10	832
Burbank	35	–	–	15,384	46	–	45	103	40	4,018	37	49	–
Carson	58	–	84	71	5,448	128	994	271	12	57	792	412	48
Cerritos	74	20	572	50	447	2,644	485	957	66	32	154	83	605
Compton	26	–	166	42	997	140	3,792	405	76	52	404	244	89
Downey	123	9	801	147	490	481	233	9,544	94	156	220	65	313
El Monte	380	266	–	123	36	56	100	88	7,283	111	32	14	40
Glendale	190	–	–	4,940	37	40	45	61	127	21,624	46	93	19
Hawthorne	–	13	–	26	512	–	215	45	28	38	4,961	1,611	–
Inglewood	–	–	14	294	298	–	426	154	28	12	1,366	5,781	16
Lakewood	45	8	1,423	61	962	1,379	521	942	16	39	245	45	3,544

Table G (continued)

Top right quarter

Long–beach	L.A.	Monte-bello	Mont-erey Park	Nor–walk	Pasa–dena	Pico Rivera	Pom–ona	Re-dondo Beach	Santa Monica	South Gate	Tor-rance	West Covina	Whit–tier
198	10,814	205	1,108	31	1,457	91	42	48	63	64	80	122	54
119	2,906	180	138	27	290	143	195	14	23	20	58	637	144
2,668	2,314	213	14	630	24	134	25	70	–	366	345	–	174
13	14,172	46	16	53	446	9	–	–	119	26	13	–	17
2,180	9,611	10	25	89	9	16	–	575	129	192	3,832	–	23
2,279	3,975	114	25	780	40	116	27	227	33	215	594	43	100
1,756	6,803	41	24	80	24	22	–	137	42	338	600	9	68
1,191	5,612	496	77	867	86	613	60	91	49	1,680	203	59	376
84	4,164	371	261	83	824	240	67	28	56	95	80	273	336
158	24,547	83	70	26	2,133	30	46	15	208	79	154	47	23
284	6,241	–	–	42	27	–	–	995	316	59	1,583	–	19
401	16,509	14	54	17	103	13	–	336	939	90	990	–	–
8004	3723	96	12	377	39	62	16	106	90	467	556	–	121

Table G (continued)

Long-beach	175	68	1366	168	4839	1457	2578	2027	222	149	780	316	3629
L.A.	2512	191	331	34, 177	6656	591	3659	2177	1700	18, 573	5492	8559	251
Monte-bello	320	52	35	93	53	27	35	175	249	140	57	74	31
Mont-erey Park	1081	27	49	121	59	79	16	193	348	177	57	88	–
Nor-walk	66	22	763	127	257	1282	418	1889	101	65	165	51	363
Pasa-dena	659	66	11	679	40	14	67	125	483	1375	10	70	25
Pico Rivera	150	26	64	38	25	123	99	805	376	109	47	–	36
Pom–ona	114	71	15	97	24	17	52	73	524	53	11	68	6
Re-dondo Beach	14	–	11	38	491	45	312	108	54	30	1494	644	11
Santa Monica	40	–	–	258	81	13	49	24	13	97	171	263	19
South Gate	55	–	86	16	302	136	487	708	31	94	116	173	89
Tor-rance	37	–	27	98	2005	71	397	200	–	38	2950	1212	50
West Covina	395	751	57	163	113	9	50	146	1589	131	11	42	48
Whit-tier	171	42	72	76	142	253	77	670	345	93	–	44	30

Bottom left quarter

Table G (continued)

67, 687	18, 687	330	189	634	114	202	102	673	315	953	3124	67	309
7784	874, 834	1601	1610	839	7783	574	434	3342	27, 518	1989	11, 452	346	867
173	7608	3710	507	63	318	694	38	–	36	87	22	28	234
141	10, 201	646	2786	74	439	169	62	56	78	28	52	90	113
1565	3363	353	12	5018	9	359	38	67	47	572	153	26	298
167	12, 448	73	319	49	21, 562	30	132	38	79	24	92	121	91
132	3975	991	268	328	64	2923	49	10	18	220	40	30	1066
139	2312	63	94	28	309	18	12, 716	–	10	90	–	412	132
610	6581	–	55	16	84	–	–	5313	151	29	4661	–	–
138	19, 729	6	15	18	96	–	–	149	15, 340	17	262	11	15
673	5824	143	73	165	11	151	33	42	37	3405	317	–	23
981	11, 875	17	16	48	59	35	41	4380	387	168	19, 919	20	46
282	7424	226	295	161	942	235	747	39	25	150	106	5082	442
468	3452	903	119	401	244	868	120	–	25	128	83	123	8310

Bottom right quarter

232

Table H: Journey to Work Data for Los Angeles County (Condensed from Table G)

To find the number of people living in area X and going to work in area Y, look for area X in the first column and area Y in the first row.

Area Key: Each of the following areas is a combination of cities listed in Table G.

1. Torrance = Torrance, Hawthorne, Inglewood, Redondo Beach
2. Santa Monica = Santa Monica
3. Carson = Carson, Compton, South Gate
4. Longbeach = Longbeach, Cerritos, Lakewood
5. Downey = Downey, Bellflower, Montebello, Norwalk, Pico Rivera, Whittier
6. Los Angeles = Los Angeles
7. Glendale = Glendale, Burbank
8. Pasadena = Pasadena, Alhambra, Monterey Park
9. Pomona = Pomona, Baldwin, El Monte, West Covina

	Tor-rance	Santa Monica	Carson	Long-beach	Dow-ney	L.A.	Glen-dale	Pasa-dena	Pomo-na
Tor-rance	58,196	1,793	5,002	2,433	826	41,206	574	449	184
Santa Monica	845	15,340	147	170	63	19,729	355	151	24
Carson	7,644	208	15,955	5,239	2,551	22,238	332	305	161
Long-beach	6,903	438	11,467	91,228	10,528	26,385	499	713	655
Dow-ney	1,947	175	6,182	10,871	50,092	26,324	1,082	2,582	2,021
L.A.	28,845	27,518	12,304	8,626	6,389	874, 834	52,750	11,905	2,671
Glen-dale	182	327	278	230	451	38,719	45,966	2,890	260
Pasa-dena	701	220	522	740	2,114	33,463	2,757	34,588	2,157
Pomo-na	543	114	793	847	3292	16,806	1445	4217	34,363

Student Handout 2-14

The L.A. Transportation Project: Making a Preliminary Plan

The Task

Your task is to design a rail transportation system for the Los Angeles County area and then present your ideas to the Los Angeles County Transportation Board (your classmates).

As a planning group you will need to make many decisions based on the data that you have already mapped out.

Here are some focusing questions to help you decide what course of action to take in designing this transportation system.

- How many rail lines will you need? Why?
- Where will you place the rail stations? Why?
- What path should each rail line travel? Why?

As you plan your system, explore different alternatives and experiment with what works best.

Presentation

Your oral presentation will be in front of the Los Angeles County Transportation Board. You have to convince the board that your plan should be used for the county's new public rail transportation system.

To convince the board, include in your oral presentation diagrams of the rail lines and rail stations and a careful explanation of the rationale for your plan. Include computations you used to make your decisions. Show how your plan makes use of these computations and the *Journey to Work* census data.

All diagrams and maps should be made large enough for viewing by all the members of the Transportation Board and the audience.

The L.A. Transportation Project
Option 1: Cost Factors

Overview

In option 1 students create and present a plan for the construction of a rail transportation system. The additional factor influencing their decisions in this option is *cost*.

Skills

estimating, balancing budgets, developing logical thinking and reasoning, converting units, working with large numbers

Materials

(for each group) Los Angeles County map and tables from the starter activity, copies of Handouts 2-15 and 2-16, calculators. Make available some of the following: oaktag, large sheets of newsprint, magic markers, meter sticks, pencils, blank overhead transparencies, acetate sheets, and dry erase markers

Time Needed

four–six class periods

Activity 1: Estimating the Cost of Construction

Use Student Handout 2-15

1. Divide students into groups and give each group a copy of Handout 2-15 and other materials. The students will use the preliminary designs that they made for the last activity (Handout 2-14).

2. The students should start with their preliminary design and calculate the costs of building their system with the information given in Handout 2-15. They will need to add trains to their system to determine these costs. Encourage students to estimate the number of trains.

3. The next step is for students to modify their original plan so that the total construction costs amount to $10 billion or less. Encourage them to try many alternatives before deciding on a final plan. Have them keep a record of the different alternatives they try so they can compare them.

   Students may need to review techniques for working with large numbers, including scientific notation, etc. If students enter printed costs into their calculators, they will often get error messages because the numbers have too many digits.

4. Finally, have the students present their design plans, together with maps, costs, and rationales, to their classmates.

The costs listed in Handout 2-15 were calculated based on the expenses of the Massachusetts Bay Transportation Authority and the Miami-Dade Transit Agency. An attempt has been made to make these costs as realistic as possible. However, for the sake of manageability the substantial cost factors of debt service, insurance, and land purchase have been ignored. Note that the costs listed on Handout 2-15 are light-rail costs applicable in urban metro systems. Amtrak or other long-distance rail costs are higher.

Activity 2: Staying within a Budget

Use Student Handout 2-16 and Table G

To further complicate matters, Handout 2-16 lists some of the major annual operating costs of an urban transit system. In this activity, your students' task is to come up with an annual operating budget of costs and revenues. They will use Table G to project numbers of riders, so they can calculate expected ridership fees.

If students have trouble making their revenues match their expenses, have them think of other possible ways to earn revenue. Note that many metro systems sell advertising space on train cars and rent tunnel space to phone companies for their cables. Also, most metro systems depend on city or state subsidies.

Student Handout 2-15

The L.A. Transportation Project: Rodent Control Costs How Much?

The Task

In this activity you are asked to design a transportation system for Los Angeles that takes into account the costs of *building* the system. The following is a list of the costs of constructing the various major components of a rail system.

Construction Costs

Components	*Cost*	*Explanation*
Train Stations	$85,000,000.00 each	(includes construction materials and labor for one station and parking lot)
Track	$260.00 per foot	(includes labor and track materials)
plus	$4.60 per foot	(includes other costs for building the tracks: rodent control, police protection, air and water monitoring systems, and ultrasonic testing of wells)
Train cars	$1,490,000.00 per car	(cars also serve as engines)
Tunnels	$10,000.00 per foot	(includes materials and labor)
Elevated structures	$2,600,000.00 per mile	
Power systems	$1,260.00 per foot	(includes substations)
Communication and signal systems	$1,180.00 per foot	

In your preliminary plan you have sketched out where you want the rail lines to be. Now decide how many cars your system will have and whether your rail lines will be below ground, at ground level, or above ground (elevated). Here are some things you need to keep in mind as you make these decisions.

Construction Notes

1. You may put in tracks at ground level only where the map shows train tracks already. This is the only land that you can assume is government owned. (The government does sometimes take land "by eminent domain," but this almost always leads to lawsuits, which you want to avoid.)

2. If you want to build below ground or above ground (not at ground level), assume that the government does not have to own the land. Thus there are fewer restrictions in building above or below ground than at ground level.

3. You may not need tunnels throughout your system. Some metro lines can be above ground if that suits your design.

4. Trains on one set of tracks must all travel in the same direction; otherwise they'll collide. The cost per foot of track listed above is for *one* set of tracks.

5. Electrical power systems and communication and signal systems must be placed wherever there is track. Otherwise the track won't be operative.

Stage 1

Determine the cost of building your preliminary plan using the figures above for the major components of your rail transportation system and following the restrictions in the "construction notes."

Here are some questions you might ask to help you perform this computation.

- How many miles of rail line have you included in your preliminary plan?
- How much does it cost per mile to construct a line (a) at ground level;(b) underground; (c) above ground level?

- How many trains did you assign to each rail line?

- How many stations are on each line?

Stage 2

In reality, communities don't have unlimited funds to build major projects like this. Assume now that the county has *10 billion dollars* to build the transportation system. Modify your original plan so that you can stay within this budget.

- You will either need to cut out some of your rail lines or cut down on the number of cars or stations per line. How will you do this?

Try various alternatives, like reducing the number or length of lines, while leaving the number of trains per line unchanged. Then vary the number of trains.

When you have found a combination that keeps you within your budget, consider the following questions:

- How many people can your new system transport on a given day? Estimate the percentage of total travelers this represents.

- Are there ways of transporting more people while keeping construction costs down?

If your design is not able to transport a reasonable percentage of travelers on a given day, continue working with different alternatives until you have a combination that seems more satisfactory.

Keep track of the different alternatives you try. Write out a description of how you came to your final plan. Include tables of computations that helped you make your decisions.

Stage 3

Present your revised transit system and budget to your classmates. Describe the major changes that you had to make, if any, to your preliminary plan in order to stay within the 10 billion dollar budget.

Student Handout 2-16

Estimating Operating Costs

Now estimate the cost of running your new system in a given year. Use the following list of operating costs. Then you will have to figure out how much to charge people to travel on your system. Detailed commuter information is given in Table G. This will allow you to estimate numbers of passengers on different lines.

Costs of Operating Transportation System Per Year

Category	*Cost*
Train crew, 2 people per train	$27 per hour, per person (includes benefits)
Station crew, 3 people per station	$24 per hour, per person (includes benefits)
Maintenance crew, 1 person per 30,000 vehicle-miles per year	$27 per hour, per person (includes benefits)
Energy costs	$1.15 per car per mile

(Note: If a four-car train goes two miles, that makes eight vehicle-miles. The maintenance crew includes carpenters, painters, emergency crews, wire crews, sheet metal workers, engine mechanics, plumbers, ventilation mechanics, and other mechanics.)

Stage 1

To estimate operating costs, use the system you designed in Handout 2-15. In addition, you will need to estimate how far your trains will travel in a year.

Stage 2

Once you have determined operating costs, think about the following questions to estimate an appropriate fare.

- How many people will ride the system per year, on average?

- Should a longer ride cost more than a short ride?
- If you wanted fares to pay for operating costs, what would you have to charge your passengers? Does this fare seem reasonable to you?

Write out a careful description of your fare plan.

Stage 3

Present your plan to your classmates.

The L.A. Transportation Project, Option 2: Traveling on Time

Overview

Users of public transportation systems are dependent upon time schedules for their traveling needs. Starting with their preliminary plan, students develop a train schedule and redesign the rail system so that people will be able to get to work on time. The lesson begins with a formula-making activity on distance, rate, and time.

Mathematical Concepts

rate-distance-time relationship

Skills

calculating, averaging, developing and using formulas, measuring distances, and weighing options

Vocabulary

maximum, capacity, schedule, intervals, destination, rate

Materials

tables from the starter activity, Los Angeles County map, group preliminary plans, calculators, stopwatches. Make available: oaktag, large newsprint, straightedge, magic markers, index cards

Time Needed

five–seven class periods

Activity 1: Distance, Rate, and Time (Formula Making)

This activity is a discussion designed to help students understand the distance-rate-time formula.

Ask either the whole class or small groups to consider the following questions. Write the first two questions from each set on the chalkboard.

Question Set A

- If you traveled 60 miles per hour (mph) for 8.5 hours, what was the distance you traveled? (510 miles.)
- If you walked 2.8 mph for 9.5 hours, what was the distance you walked? (26.6 miles.)
- In the first two questions, what did you do to come up with your answers? (Students have multiplied mph, the *rate*, by the *time*, to find their answers.)

Now they have found *distances*.

Question Set B

- The distance from Boston to New York City is 220 miles. If the bus you were riding took 5.5 hours to travel from Boston to New York City, then, on average, how fast did the bus travel, or what was the bus's rate? (40 mph.)
- Let's say that you took a train from Boston to New York City, and the travel time was 6.25 hours. On average, at what rate did the train travel between the two cities? (35.2 mph.)
- In the last two questions, what did you do to come up with your answers? (Students have divided the *distance* by the *time* to find their answers.)

Now they have found *rates*.

Question Set C

- A car averaged 55 mph and traveled 495 miles. How long (time) was the car driven? (9 hours.)
- An airplane averaged 610 mph and traveled 4,575 miles. How much time did the plane ride take? (7.5 hours.)
- In the final two questions, what did you do to come up with your answers? (Students have divided the *distance* by the *rate*.)

Now they have found *time*.

Next, by using variables to represent the numbers in each question, have students create a formula for each of the three situations A, B, and C above. Give your students the time and encouragement to form their own basis of understanding. Examine the student-created formulas for correctness.

Now offer students the letters: D for distance, R for rate of speed, and T for time. If they used other symbols in their formulas, ask them to write new formulas using these letters.

Ask the students what similarities and differences they notice in their three formulas. Encourage them to realize that the formulas use the same symbols but are rearranged and sometimes use different mathematical operations. In the first group of questions, multiplication is used ($R * T = D$). In the last two groups of questions, division is used ($D/T = R$ and $D/R = T$). Ask students if they see any patterns in the arrangement of the last two equations. (Distance is always being divided by another variable.)

Put the three equations on the board and ask the students how the equations are related to each other. (They can all be derived from the first equation by dividing both sides by either time (T) or rate (R).)

This is the standard formula describing the relationship between distance, rate, and time: $D = R * T$ in simple linear motion.

When students are faced with problems involving simple linear motion, they are often given numerical values for two of the variables. They will then have to decide how to arrange the variables in order to find the value of the third.

Activity 2: Pacing against Time

Use Student Handout 2-17

Tell students that they will be conducting an experiment with the distance-rate-time formula. Divide students into groups of three.

1. Tell each group to measure off a distance of 100 feet. Give each group a stopwatch. One person will be the starter, the second person will be the traveler, and the third person will be the timer.

2. Remind students that there are three variables in the distance-rate-time formula. Ask them, in setting up the experiment, which variable(s) do they now know? (Distance.)

3. Each group should conduct the exercise as described on Handout 2-17.

This activity offers students two insights. First, they will experience the distance-rate-time formula kinesthetically. Second, they will understand that during a given trial, rate varies and that the R in the formula is really an *average* rate.

The next activity builds on this one and brings in the idea of a function.

Activity 3: Graphing Time and Rate vs. Distance

Use Student Handout 2-18

Have your students graph the results of their field tests using Handout 2-18. They may use either bar graphs or line graphs. When everyone is done, discuss the different kinds of graphs each group made. Pose the following questions:

- Which type of graph did you choose and why?
- What observations can you make from your graphs?
- Do different graphs show different information?

These graphs are helpful in understanding the concept of a function. If y is a function of x, then for every x value, there is a single corresponding y value. At each 10-foot interval, there is one corresponding time measurement. For example, at the first 10-foot interval, the timer may record the pacing as taking 5 seconds. At the second 10-foot interval, the timer may record the pacing at either 8 seconds (total) or 3 seconds between the first and second interval. In either case, for each interval of distance (x value), there is a single corresponding time (y value). The graph helps students see this more clearly.

Activity 4: Traveling on Time

Use Student Handout 2-19 and Table G

Students now have the opportunity to use the formula $D = R * T$ to assist them in formulating a time schedule for their rail system. Notice that the activity gives four assumptions, including a time frame during which the passengers must be transported so that they arrive at work on time.

In designing a time schedule and calculating times, rates, and distances, students probably need to make significant adjustments to their preliminary rail system. Make copies of Table G available to each group. Encourage students to take one line at a time, and ask them to check their calculations carefully. They will produce a revised design.

Have students present their revised transit system plans to their classmates and explain any significant changes they needed to make in order to accomodate ridersip.

Going Further

The following activity can be given to small groups or to individuals as a homework assignment: Ask students to make a time schedule on index cards for each train station. Use times between 6:15 a.m. and 9:00 a.m.

Have the groups trade their time schedules and system designs with other groups. Each group should choose a line to check, and make sure that 78 percent of the travelers on that line can get from home to work on time.

Student Handout 2-17

Pacing against Time

In this activity you will experiment kinesthetically (through body movement) with the distance-rate-time formula. Consult with your teacher about how to divide up tasks between the members of your group.

1. Measure out and mark a distance of 100 feet. Record the process you used for measuring the distance and why you used that process.

2. Begin the experiment by having one member of your group travel the distance you measured out, and use a watch or stopwatch to find the time it takes to travel that distance.

   - Time for initial trip:

   - Record how it felt to travel this distance in this amount of time.

3. Now have someone in your group travel the same distance in each of the following specific time intervals. (In other words, time the trip so you take exactly this amount of time from start to finish.)

   a. 15 seconds

   b. 48 seconds

   How did it feel to travel the distance in this amount of time?

4. Now, try traveling the same distance using each of the following specific *rates*.

   a. 14 feet per second

   b. 6 feet per second

   c. 2 feet per second

   Record the time it took to travel the distance at each of these rates. Also record how it felt to move at these rates.

   Is there a way to record these times without having to actually walk the distance? If so, explain why.

5. In each of the last two steps of the experiment, which quantities *vary*? Which quantities remain *constant*?

6. Summarize what you have discovered about the formula D = R * T.

Student Handout 2-18

Graphing Time and Rate

Now mark off the 100 feet in ten-foot intervals.

1. Travel the distance in forty seconds, and time how long it takes to travel *each interval of ten feet*. You need to think carefully about how you want to do this. Use the following table for recording your times.

Distance (in feet)	*Time for that interval*	*Cumulative time*	*Rate*
10			
20			
30			
40			
50			
60			
70			
80			
90			
100			

2. Now make a graph of the *time* for each interval as a function of distance. Choose any type of graph that seems appropriate.

3. Calculate the *average time* it took to travel one ten-foot interval.

4. In a different color on the same graph show the average time it took to travel one ten-foot interval, so you can compare the average time with the actual times for each interval.

5. Do steps 2–4 for *rate* instead of time. You should first graph the rate for each time interval, then calculate the *average rate* for the whole trip and mark the average on your graph for comparison.

6. What do you notice about average rates and actual rates?

Student Handout 2-19

Traveling on Time

After completing the preliminary plan in Handout 2-14, you are now ready to work on a time schedule for your rail system.

Assume the Following Facts

1. 78% of the people on your subway lines need to arrive at work by 9:00 a.m.

2. Each subway car has a seating capacity of 54. In addition each subway car has standing room for 126 people.

3. Each subway train consists of a maximum of 6 train cars. Remember, each train car can run by itself.

4. A train's average speed is:

   40 miles per hour, above ground
   15 miles per hour, at ground level
   25 miles per hour, below ground

Revising your Preliminary Design

Using the above assumptions, recreate your rail system design, factoring in the variable of time.

Questions to consider when planning your revised rail system:

1. How many people (in total) travel to work on each subway line in one working day?

2. How many people will need to be transported between the hours of 6:15 a.m. and 9:00 a.m.?

3. How many trains are necessary to transport all these people to work? How did you go about finding this figure?

4. Say the first train leaves the station at one end of the line at 6:15 a.m. At what time intervals would the subsequent trains leave the same station, so that every person arrives at work by 9:00 a.m.? Describe how you solved this problem.

Appendices

Appendix A: NCTM Goals

The following chart correlates the activities in *The Power of Numbers* with the standards outlined by the National Council of Teachers of Mathematics in their *Curriculum and Evaluation Standards for School Mathematics* (1989). The standards listed are those for grades 5–8.

Part 1

Activity \\ Standard	Math as problem solving	Math as communication	Math as reasoning	Mathematical connections	Number & num. relationships	Num. systems & num. theory	Computation & estimation	Patterns & functions	Algebra	Statistics	Probability	Geometry	Measurement
Monitoring the Future		•	•	•						•			•
Bias in context		•	•	•									
Mechanics of questions			•	•									
Populations & samples		•	•	•						•			
Sampling experiment	•	•	•	•	•		•			•	•		•
Fishcounting	•	•	•	•	•		•	•	•	•	•		•
Randomness				•		•		•		•			
Random sequences	•			•		•		•		•			•
Three experiments	•		•	•		•				•			•
Digits of π, $\sqrt{2}$, & 5/17	•		•	•		•	•	•		•			•
Going further			•	•		•		•	•	•			•
Random samples			•	•		•		•		•			
Pick a sample		•	•	•						•			
Designing a poll		•		•						•			•

	Math as problem solving	Math as communication	Math as reasoning	Mathematical connections	Number & num. relationships	Num. systems & num. theory	Computation & estimation	Patterns & functions	Algebra	Statistics	Probability	Geometry	Measurement
Introduction of averages		•	•	•			•			•			•
Averages & freq. dist.				•						•			•
Mean average	•	•	•	•	•		•			•			•
Diversity in School Pop.	•	•	•	•	•		•						•
Neighborhood restaurant	•	•	•	•	•		•						•
Average arbitrator	•	•	•	•	•		•						•
Line graphs	•	•	•	•	•		•						•
Circle graphs	•	•	•	•	•		•						•
Bar graphs	•	•	•	•	•		•						•
Picture graphs	•	•	•	•	•		•					•	•
Graphing pop. density	•	•	•	•	•	•	•						•
Comparing graphs	•	•	•	•	•								•
Scaling graphs	•	•	•	•	•								•
Use averages & graphs	•	•	•	•	•		•						•
Intro. to interpretation			•	•						•			
Meaning in percentages			•	•	•		•			•			•
Portrait of a generation		•	•	•			•		•				•
Opposing point of view		•	•	•									•
Margin of error		•	•	•	•		•			•	•		•
Error in TV samples			•	•	•		•			•	•		•
Final presentations	•	•	•	•									•

Activity \\ Standard	Math as problem solving	Math as communication	Math as reasoning	Mathematical connections	Number & num. relationship	Num. systems & num. theory	Computation & estimation	Patterns & functions	Algebra	Statistics	Measurement
Taking a class census	•	•	•	•	•		•			•	•
History & controversy		•	•		•		•				
Estimating enumerators	•	•	•	•			•				•
Go further: school funds	•	•	•	•							
Census, reapportionment	•	•	•	•	•		•	•			•
Average district size	•	•	•	•	•		•				
Reapportion in Frachat	•	•	•	•	•	•	•	•			•
... by Webster method	•	•	•	•	•	•	•	•	•		•
... by Hamilton-Vinton	•	•	•	•	•	•	•		•		
Comparing two methods	•	•	•	•	•	•	•	•	•	•	•
Go further with Webster	•	•	•	•	•	•	•	•	•		•
Effects of undercounting	•	•	•	•	•		•				•
Redistricting in Billows	•	•	•	•			•				•
Districts in your state	•	•	•	•			•				•
Rates of change: pop.	•	•	•	•		•	•	•	•		
Find aver. rates of change	•	•	•	•	•	•	•				
Recognizing patterns	•	•	•	•	•	•	•	•	•		
Formula making	•	•	•	•		•	•	•	•		•
Graphing rates of change	•	•	•	•	•		•	•	•		
Rates & trends over time	•	•	•	•	•		•	•			

	Math as problem solving	Math as communication	Math as reasoning	Mathematical connections	Number & num. relationship	Num. systems & num. theory	Computation & estimation	Patterns & functions	Algebra	Statistics	Measurement
Doubling time	•	•	•	•	•		•	•	•		•
React to exponential ch.	•	•	•	•	•		•	•	•		
Doubling time - U.S. pop.	•	•	•	•	•		•	•	•		•
Cross referencing	•	•	•	•	•		•	•	•	•	•
Mapping it out	•	•	•	•							
Develop preliminary plan	•	•	•	•				•			
Construction costs	•	•	•	•	•		•	•		•	•
Staying within a budget	•	•	•	•	•		•	•	•	•	•
D-R-T formula making	•	•	•	•			•		•		
Pacing against time	•	•	•		•		•	•			•
Graphing T, R, & D	•	•	•	•	•		•	•	•		•
Traveling on time	•	•	•	•	•		•		•		•
Final L.A. presentation	•	•	•	•	•		•	•	•	•	•

Appendix B: Cooperative Learning Techniques and *The Power of Numbers* *

Most of the activities in *The Power of Numbers* can and should be done by students in cooperative learning groups. Students will see much more in the material, particularly where complex or subtle concepts are involved, if they are exposed to different approaches and views. Further, they learn much more about communicating in mathematics and social studies when they work with other students.

Productive, effective, cooperative learning in small groups is not automatic, however. Below are a number of techniques that may be helpful throughout the curriculum. Following this brief listing is a short bibliography of resources with many more ideas.

Getting into Groups

Most of the activities in *The Power of Numbers* can be done in groups of three or four students. In general, groups of that size provide a balance for two competing goals: groups are large enough to stimulate a diversity of ideas and are small enough so no student is lost in a crowd. Where an activity is best done by a different number, it is explicitly stated. For example, in the sampling experiment in Part 1 pairs of students work best.

If you want groups to be organized by a random process, there are many techniques that can be used. For example, have students line up by a specific criterion—birthdays, alphabetical by name, etc.—and then split the line where useful. Students will have to be especially creative and cooperative if they have to line up without talking.

The following method can also be used if you want randomly organized groups. Shuffle and then hand out cards, each with a mathematical expression on it, and have students find group partners with expressions that are equivalent. For example, the student who has a card saying "2 x 6" would have to find the student with a card saying "3 x (1 + 3)" or any other expression equivalent to 12. This can also be done with picture pieces that fit together or sections of well-known quotations, etc.

If you plan to assign groups, it is usually more effective to mix students with difficult behaviors into different groups, rather than grouping them together. Also, mix students according to characteristics relevant to their projects, such as social skills, verbal skills, or the ability to think critically, to problem solve, to organize, etc. Many activities can take advantage of diverse abilities. For example, a student group has chosen a topic for its community poll and has decided to conduct interviews to gather data (Part 1). That group will need members who have patience and perseverance, like to talk to new people, can write interesting questions, will determine the best locations and times to find people, can analyze data, like to write reports, and can contribute in many other ways.

*We are grateful to Ellen Davidson for her assistance. Many of the techniques listed were adapted from her "47 Good Ideas."

Helping Small Groups Work Well

Use team-building activities, cooperative games, or initiative games as necessary to build various group-processing skills. (See the bibliography at the end of thissection for suggested resources.)

If there are only two students per group, have them face the same direction so they can both read anything necessary. If small groups have three or more students each, make sure students are able to sit close enough to each other to hear and be heard easily and to participate fully.

Have students discuss and set ground rules that will help them work more effectively, such as rules about respect, listening, and how to handle disagreements. How would each of them like to be treated in their groups? One guideline might be "Let each person finish what he or she is saying before the next person starts." Include a ground rule covering positive and negative expressions about group assignments. Some ground rules may take practice before students can implement them. Learning how to disagree might take a few role-plays and discussions. If students are having trouble listening to each other, try a guideline such as "Someone has to paraphrase the last idea spoken before a new one can be offered."

If some students participate much more than others in the groups, distribute talking tokens (beads, beans, cards, any small item). Give each member of the group the same number of tokens at the beginning of the activity. Each time a student contributes an idea, she or he has to place a token in a cup in the center of the group. When all a student's tokens are used up, she or he can not say anything until everyone has used all their tokens as well. Then redistribute the tokens.

For some activities, have students work alone at first, writing down some thoughts. This can be helpful for giving all students the chance and the responsibility to think and prepare.

Encourage experimentation in combinations of independent work and group work. Some groups may find it helpful to start with silent independent time; others may work best by brainstorming and organizing first and then dividing up tasks for independent work to be done afterward. There are many effective combinations.

If cooperative groups are new to students, have them fill out short group evaluations with such questions as: What worked well today? What do we need to improve? Students should need these evaluations less as they get more experienced in group-process. Coach specific students on their group-process skills when necessary.

For Discussions or Presentations with the Whole Class

Early in the year use activities that pull the whole class together, increasing trust, communication, and a sense of interdependence.

Have students apply the small-group ground rules to the whole class, changing or adding to them as necessary.

Building Interdependence and Accountability

Have one recording sheet per small group. Have all group members sign their group's sheet when the activity is over, signifying that they understand and agree with everything on the sheet. (Make copies of these sheets for each student's folder.)

Help students delegate tasks. For example, when working on the population picture graph (Part 1), one student might be measuring, another might be calculating, and another recording. In activities involving cooperative learning groups, it can be helpful to assign roles, such as time-keeper, materials person, scribe, etc.

Evaluate the process as well as the content. Have students write in their journals about their own and other group members' participation, as well as about the topic on which they are working. Or use the short evaluation sheets mentioned above with process-oriented questions.

Assign the role of presenter *after the activity* is completed to keep all students alert. Some teachers role a die after an activity to select by a random process who will present to the whole class. There are potential problems with random selection. It could make some students anxious throughout the class, afraid that they will be chosen; and it could lead to a poor presentation that is not instructive to other students.

If you give students skills quizzes, let them choose to work alone or with other students. Students will likely refuse to work with unprepared classmates, so anyone who wants to work with others is motivated to be prepared. Students may carry a friend once, but rarely do so twice.

Have students in a group agree on questions for the teacher, with all group members raising a hand. This not only encourages students to communicate with each other and be efficient with the teacher's time, but also gives them the chance to answer each other's questions without relying on the teacher. In "Estimating Enumerators" (Part 2) allowing students only a certain number of questions per group means they have to negotiate which are the most important and most potent questions.

Cooperative learning does not preclude individual assessment. Teachers will want to know frequently how each student is doing. See the section "Assessment and Evaluation Ideas for *The Power of Numbers*" for more ideas on assessment techniques, which, without the pressure of grades, do not undermine the groups.

Experiment with weighing grades for individual and group work. If students' entire grade is dependent on group work, teachers may have to deal with the problem of noncontributing students. Conversely, if most of the grade depends on individual accomplishments, which foster competition, a group's cooperative process can be undermined. Weighing group work 75 percent and individual work 25 percent might foster both individual accountability and interdependence. Other variations might work as well. More ideas about evaluation are included in "Assessment and Evaluation Ideas for *The Power of Numbers*."

Selected Bibliography for Further Reading on Cooperative Learning

Brubacher, Mark; Ryder Payne; and Kemp Rickett. *Perspectives on Small Group Learning: Theory and Practice,*. Oakville, Ontario, Canada: Rubicon Publishers, Inc., 1990.

Clarke, Judy; Ron Wideman; and Susan Eadie. *Together We Learn*. Englewood Cliffs, N.J.: Prentice-Hall, 1990.

Cohen, Elizabeth. *Designing Groupwork: Strategies for the Heterogenous Classroom*. New York: Teachers College Press, 1986.

Davidson, Ellen, and Nancy Schniedewind. *Cooperative Learning, Cooperative Lives*. Dubuque, Ia.: W.C. Brown, 1987.

Davidson, Neil, ed. *Cooperative Learning in Mathematics: A Handbook for Teachers*. Reading, Mass.: Addison Wesley, 1990.

Gluegelman, Andrew, ed. *The New Games Book* and *More New Games*. Garden City, N.Y.: New Games Foundation and Doubleday, 1976, 1981.

Johnson, David W.; Roger T. Johnson; Edythe Johnson Holubec; and Patricia Roy. *Circles of Learning: Cooperation in the Classroom*. Alexandria, Va.: Association for Supervision and Curriculum Development, 1984.

Rohnke, Karl. *Silver Bullets*. Dubuque, Ia.: Project Adventure, Kendall/Hunt Publishing Co., 1984.

Appendix C: Assessment and Evaluation Ideas for *The Power of Numbers*

Both assessment and evaluation are attempts by teachers to find out what students know and what they can do, though distinguishing between the two processes are important.

In his book *Teaching Ten to Fourteen Year Olds* Chris Stevenson gives clear definitions of the two procedures. "Assessment," he writes, is "the process of collecting information that helps students and teachers understand how students are doing. Student journals that are shared with teachers are a commonly used assessment device. Written anecdotes enable the student to both clarify an issue or idea for himself or herself through writing and at the same time convey it to a teacher." Stevenson adds: "Evaluation: the process of employing selected, specific criteria for the purpose of judging and critiquing the quality of a performance, production, or other evidence of learning. A familiar example of evaluation is a spelling test composed of words drawn from classwork. This device evaluates students' knowledge of correct spelling for selected words." [1]

Traditional evaluative measures, such as standardized tests, are not very informative. In the past, teachers generally used a timed paper-and-pencil test, and school systems used timed standardized multiple-choice tests for evaluating student performance. These tools give only one snapshot of a student's abilities, and usually only those abilities that are easy to measure. They also have many well-documented negative effects on self-confidence and on students' appreciation of the value of and relationship among failure, success, and learning. Teachers need other measures to better understand how their students are processing information and applying skills. Hence, there has been an attempt to move toward alternatives and away from paper-and-pencil tests and standardized tests.

Below are examples of assessment techniques and ideas for evaluating students that may be helpful when you use *The Power of Numbers*. Additional assessment tools include observations and interviews. The line separating one type of assessment from another is not always clear, nor is it always important. A single activity may be an investigation, a performance task, and an open-ended question all at once.

In addition to these assessment techniques, it is crucial for teachers to refine their skills as keen observers and thorough recorders. Teachers may want to develop new recording sheets. They might include such headings as strategies used to solve problems, materials or tools used to solve problems, communication of comprehension, or illustrations of the problem. Other headings may include group interactions, checking, encouraging, or summarizing. So much of assessment is dependent upon what teachers see, hear, and understand. Paying attention to these skills affords classroom teachers a better understanding of their students, while also enhancing their ability to report on the progress of students to the students and their parents.

---

[1] Samuel Christopher Stevenson, *Teaching Ten to Fourteen Year Olds* (New York: Longman Publishing Group, 1992: 246).

Following the assessment and evaluation ideas is a short bibliography for further reading. There is a great deal of literature on alternative assessment, with many suggestions for scoring criteria, design of performance tasks, etc. We encourage teachers to explore these resources.

Performance Assessment

"Performance assessment involves giving a group of students, or an individual student, a mathematical task that may take a half an hour to several days to complete or solve. The object of the assessment is to look at how students are working as well as at the completed tasks or products."[2]

"Looking at student performance gives information about their ability to:

- reason soundly and raise questions
- persist, concentrate, and work independently
- observe, infer, and formulate hypotheses
- think flexibly, changing strategies when one doesn't work
- use manipulative materials, equipment, calculators, and computers
- work together in groups
- communicate and use mathematical language through discussing, writing, and explaining their ideas in their own words
- use estimation
- detect and use patterns
- design and conduct experiments and investigations
- collect, organize, and display information
- get excited about mathematics." [3]

---

[2] *Assessment Alternatives in Mathematics*, by Jean Kerr Stenmark, EQUALS and the Assessment Committee of the California Mathematics Council Campaign for Mathematics, University of California, Berkeley, Calif., 1989, p. 20.

[3] Ibid p. 20.

The Power of Numbers contains many activities that can serve as performance-assessment opportunities. Students demonstrate to what degree they can apply and explain how and why they used certain skills and what results they produced. Teachers can understand more about their students' thinking, whether or not students have internalized skills and connected them to a larger body of knowledge and experience. A few of the activities that can serve as performance assessments are:

- Sampling Experiment
- Going Further with Sampling: Fishcounting
- Graphing Population Density (in "Experimenting with Graphs")
- Welcome to the State of Billows (in "Redistricting")
- Formula Making (in "Rates of Change")
- L.A. project

Portfolio Assessment

"The portfolio is a new and powerful method of assessing students' mathematical learning. An assessment portfolio is a planned selection of students' work collected from the work done throughout the school year. Portfolios should include products that reflect a student's developing mathematical power. An assessment portfolio, however, is not the same as a student's folder. A student's folder may contain all of the student's work, whereas the portfolio contains a carefully chosen sample.

"A portfolio should contain typical samples of work as well as the best efforts of the student. A 'typical sample' should show evidence of growth in learning—to what degree the student has developed mathematical power."[4]

Throughout *The Power of Numbers* teachers and students can collect writing samples or examples of constructions that illustrate an individual's or group's progress. Some of the activities that can be used for this process are:

- Polling project
- Meaning in percentages (in "Interpreting Polls")
- Experimenting with Graphs, all activities
- Formula-Making Activity (in "The Census and Reapportionment")

---

[4] *A Sampler of Mathematics Assessment*, California Assessment Program, California Department of Education, 1991, p. 25.

- Welcome to the State of Billows (Scenario 3 in "Redistricting in Billows")
- Doubling Time of the U.S. Population
- Extrapolating New Data (in "Cross-Referencing")
- Student record of mathematical vocabulary

Presentations

In doing presentations, students can show the teacher and their peers that they have accomplished a task, have created a product, or can demonstrate a concept. Presentations can be small progress reports or major concluding events. They automatically carry an added benefit; that is, they not only provide a vehicle for assessing what a student or group of students have accomplished, but also continue the education process as students learn from each other. There are many activities and projects in *The Power of Numbers* that call for student presentations, including:

- Neighborhood Restaurant (in "From Highs to Lows: An Average Tour")
- Polling projects
- Rates of Change, graphing activity
- Los Angeles transportation project

Student Self-Assessment

Through journals or self-assessment questionnaires, students can show changes in attitudes about the curriculum content or about their abilities, connections that they are making between curriculum materials and the outside world, questions that currently interest them, and much more. The journal also gives them a forum for reflecting on their social skills, such as their group participation or confidence in front of the class.

Self-assessment is an important learning tool, encouraging responsibility and honesty. As students privately compare themselves to their own past performances or to their peers, they establish new challenges for themselves.

Investigations

Investigations "present students with a situation, allow for multiple entry points and multiple paths to a solution, and provide opportunities to communicate to an audience."[5]

---

[5]Ibid, p. 17.

The Power of Numbers has many activities that stress investigations that enhance students' learning. Some of these activities are:

- Which School for You? (in "From Highs to Lows: An Average Tour")
- Average Arbitrator Activity (in "From Highs to Lows: An Average Tour")
- Comparing Graphs (in "Experimenting with Graphs")
- Going Further with Federal Funds Allocated to Schools (in "Census History")
- Rates of Change, all activities

Open-Ended Questions

"An open-ended question has many avenues of access and allows students to respond in a variety of ways.

"There is a distinction between open-response questions, in which the test writer sets forth a problem or a set of steps leading to a particular answer, and open-ended questions, in which the student may give a variety of successful responses. We may choose to evaluate the diverse paths taken rather than the answers themselves."

"The important quality in an open-ended question, as in other forms of assessment..., is that we are able to see student thinking rather than test-writer thinking."[6]

Many of the activities in *The Power of Numbers* lend themselves to being open-ended. We encourage teachers to take notice of the many ideas generated by their students. Some activities that offer students multiple paths to a solution are:

- Length of the School Year (in "The Art of Questioning")
- Mean Average Activity
- Opposing Points of View (in "Interpreting Polls")
- Polling project
- Estimating Enumerators
- L.A. transportation project

---

[6] *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions*, NCTM, 1991, p. 20.

Evaluation

There is currently a great deal of controversy about evaluation. Standardized tests rarely measure students' abilities to problem solve, communicate, interpret, or demonstrate many other crucial skills. Competitive, time-limited, closed-book tests require memorization and undercut cooperative learning groups. Neither of these tend to offer students opportunities to continue learning, nor do they show the teacher how students are processing information or applying skills. However, evaluation is not limited to testing.

Edward Chittenden writes: "In its derivation, the word assess means 'to sit beside, to assist the judge.' It refers to a process of collecting and organizing information or data in ways that make it possible for people—teacher, parents, students—to 'judge' or evaluate."[7] In this light evaluation is a later stage of assessment, when criteria for judging are applied. The question then is how to make the evaluation process, that is, the choice and managing of tasks and criteria for judging, more constructive than traditional tests.

Tools for more effective evaluation vary widely. Teachers may use individual or group learning contracts, group or individual self-evaluation forms, or scoring rubrics. They may include any of the methods described above as well. Or they may use new performance tasks designed to measure a student's mastery of certain skills. For example, after the polling projects are complete, teachers might ask students to interpret articles from recent newspapers so that they can demonstrate a level of insight into polls that they did not have earlier.

There are a number of techniques that can make evaluation for the sake of grading more informative than traditional time-limited, closed-book, competitive tests. For example, if students can use notes, books, and calculators during an evaluation exercise, they will be able to demonstrate skills other than memorization. They will also be participating in a task that much more closely resembles the challenges they will face in adult work situations. The ability to use resources effectively will be important to students throughout their lives.

An important underpinning of constructive evaluation is that students know ahead of time what the criteria will be. "All real-world performers know the target and the standards, not just their task in advance; such knowledge guides their training and rehearsals."[8] In the workplace the lack of understood goals and schedules would be a sign of bad management. Some teachers may want to use the "Checklist for Completing Your Poll" as part of a learning contract for Part 1. Teachers and students can agree upon, and renegotiate if necessary, a schedule for accomplishing tasks. A class can also discuss specific criteria. For instance, are there a minimum number of questions to be asked in the student polls and should the questions be of different types? Should students use at least one of each type of graph in displaying their data?

---

[7]Edward Chittenden, "Authentic Assessment, Evaluation, and Documentation of Student Performance," *Expanding Student Assessment*, ed. Vito Perrone (Alexandria, Va.: Association of Supervision and Curriculum Development, 1991): 26.

[8]Grant Wiggins, "Creating Tests Worth Taking," *Educational Leadership* 49: 8, (May 1992):29.

Whatever method is chosen, the most important criteria for designing assessment and evaluation tools are that the tools assess the important skills and knowledge acquired, that students learn through the process, that the process helps students to enjoy learning, and that the tools help teachers, parents, and the students themselves to improve the learning process.

Selected Bibliography for Further Reading on Assessment and Evaluation

From the National Council of Teachers of Mathematics, Reston, Va.:

Arithmetic Teacher 39:9 (May 1992), focus issue on testing.

Arithmetic Teacher 39:6 (February 1992), focus issue on assessment.

Curriculum and Evaluation Standards for School Mathematics, 1989.

Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions, ed. Jean Kerr Stenmark, 1991.

Mathematics Teacher 85:8 (November 1992), focus issue on assessment.

From the Association for Supervision and Curriculum Development, Alexandria, Va.:

Educational Leadership 49:8 (May 1992), focus issue on performance assessment.

Expanding Student Assessment, ed. Vito Perrone, 1991.

A Practical Guide to Assessment, exec. ed. Ronald S. Brandt, 1992.

Davidson, Ellen, and Nancy Schniedewind. *Cooperative Learning, Cooperative Lives*. Dubuque, Ia.: W.C. Brown, 1987. (See section entitled, "Evaluation Formats for Cooperative Learning.")

Stenmark, Jean Kerr. *Assessment Alternatives in Mathematics*. EQUALS and the Assessment Committee of the California Mathematics Council, University of California, Berkeley, Calif.

Appendix D: Fitting *The Power of Numbers* into Your Curriculum

Teachers may want or need to fit *The Power of Numbers* into a particular amount of time. Using the entire curriculum would take at least one hundred class periods, given a typical forty five-minute period. Teachers who wish to use *The Power of Numbers* for a shorter period of time will have some choices to make and will need to assign more activities as homework, but students should still be able to experience and experiment with mathematics in context.

We offer below *suggested formats* for this curriculum. Depending on your students, what skills they have already learned, what skills you want to emphasize, etc., you can, of course, alter these formats. If you plan to use *The Power of Numbers* in conjunction with a traditional skills book, you will need to add time or alter the formats below.

If you have limited time, your first choice is which part to use. If you have only three to six weeks, students will get much more out of completing a few activities or projects in depth than several cursorily. We recommend using a month or more for Part 1 on polling, three weeks or more for Part 2 without the Los Angeles transit project, and three weeks or more for the Los Angeles transit project without the rest of Part 2. Within the Los Angeles transit project there are two options following the preliminary plan. You may need to choose just one of them.

If you use Part 1, please note that students design and create their polls throughout the first half. They then conduct their polls by distributing a questionnaire or conducting interviews or using other methods. Whatever method they choose, they can collect the data outside class time (or wait for questionnaires to be returned) while doing the activities in the sections on averages and graphs.

There are at least *two important pedagogical concerns* to consider if you plan to use only sections of *The Power of Numbers*. First, whether you use this curriculum in a mathematics or a social studies class, you will have to deal with students' expectations of what it is (or has been) like to use math skills. Through the course of this curriculum and others like it, students learn some habits of mind that are different from traditional mathematics applications. They learn that math "answers" can be preliminary and can be reworked. They learn that there can be several appropriate answers as well as several different questions that can be raised. They learn that mathematics used in a real-world context can prolong a problem into days and weeks, rather than just minutes. If you have students do, for example, just the Los Angeles transit project, while they may have all the computation skills necessary, they may not yet have built the necessary habits of mind.

Second, many activities can, and indeed should, be conducted in cooperative-learning groups, particularly those activities involving subtle and complex material. By limiting the time and moving some activities to homework assignments, you may make it harder for students to participate in cooperative groups, especially if they commute to school from different neighborhoods.

In the following charts a "•" suggests that you include this activity. Depending on the needs and abilities of your class, you may have them do the activity in one or several class sessions, as homework, or both. Activities that may be omitted are designated with a dash.

Part 1

Activity	1 month	6 weeks
A Poll about Young People: Activity 1	•	•
Activity 2	fold into act.1	•
What Does This Have to do w/ Math? Social Studies?	•	•
Choosing a topic	•	•
The Art of Questioning: Activity 1, Activity 2	•	•
Activity 3	•	•
Activity 4	–	–
Activity 5	•	•
Nuts and Bolts of Statistics: Activity 1	•	•
Activity 2	•	•
Fishcounting	–	–
Activity 3	–	•
Activity 4	•	•
Activity 5	•	•
Design and Conduct	•	•
From Highs to Lows: An Average Tour: Act. 1	–	•
Activity 2	•	•
Activity 3	–	–
Activity 4, Activity 5	•	•
Activity 6	–	–
Experimenting with Graphs: Activity 1	•	•
Activity 2, Activity 3	•	•
Activity 4	–	•
Activity 5	–	–
Activity 6, Activity 7, Activity 8	•	•
Interpreting Polls: Activity 1	•	•
Activity 2	–	–
Activity 3	•	•
Activity 4	–	•
Activity 5	•	•
Final Presentations	•	•

Part 2

Activity	3 weeks	6 weeks
Classroom Census	–	•
Census: History and Controversy: Activity 1	•	•
Sample Questions from the Census	fold into act.1	fold into act.1
First Census of the U.S.	fold into act.1	fold into act.1
Activity 2	•	•
The Census and Reapportionment: Activity 1	•	•
Activity 2	•	•
Activity 3	•	•
Activity 4	•	•
Activity 5	–	•
Activity 6	–	•
Activity 7	•	•
The Census and Redistricting: Activity 1	•	•
Activity 2	–	•
Rates of Change: Activity 1	•	•
Activity 2	•	•
Activity 3	fold into act.2	•
Activity 4	•	•
Activity 5	–	•
Activity 6	–	•
Doubling Time: Activity 1	•	•
Activity 2	–	•
Activity 3	•	•
Cross-Referencing Census Data: Activity 1	•	•
Activity 2	•	•
Activity 3	–	•

Los Angeles Transportation Project	3 weeks	4 – 5 weeks
Creating a Public Rail Transportation System for Los Angeles County: Mapping It Out	•	•
Developing a Preliminary Plan	•	•
* Option 1: Cost Factors: Activity 1	• –	•
Activity 2	• –	•
* Option 2: Traveling on Time: Activity 1	– •	•
Activity 2	– •	•
Activity 3	– •	•
Activity 4	– •	•
Going Further	– •	–
Final Presentations	• •	•

* Choose Option 1 *or* Option 2 if students have only three weeks for the Los Angeles transit project, or have some students work on Option 1 and others work on Option 2. There will not be enough time for everyone to do both.

Appendix E: Be Prepared with Special Materials

Most of the activities in *The Power of Numbers* require standard school supplies: handouts, paper, rulers, calculators, graph paper, etc. There are, however, several activities that use materials not so common in schools and that teachers will have to acquire or collect. Here is a list of those activities and materials.

Part 1

Nuts and Bolts of Statistics
- Activity 2: many decks of card or beans or beads
- Activity 2 variations: cardboard pieces, or milk jug lids and yogurt lids, or other objects varying in size, thickness, or shape

Designing and Conducting Polls: classes might need phone books or class lists

From Highs to Lows: An Average Tour
- Activity 3: "Help Wanted" sections from newspapers

Experimenting with Graphs
- Activity 5: encyclopedia or world almanac
- Activities 5 through 8: optional use of spreadsheet software and computers

Part 2

The Census and Reapportionment
- Activity 2: optional use of spreadsheet software and computers
- Activity 3: pennies, tiddlywinks, or other small objects
- Activities 4 through 7: optional use of spreadsheet software and computers

The Census and Redistricting
- Activity 2: map of congressional districts for your state

Doubling Time
- Activity 2: nerf balls or other soft objects, such as sponges

Los Angeles Transportation Project
- several maps of Los Angeles County

References

Bachman, Jerald G., Lloyd D. Johnston, Patrick M. O'Malley. *Trends in Drug Use and Associated Factors among American High School Students, College Students, and Young Adults: 1975–1989. From Monitoring the Future: A Continuing Study of the Lifestyles and Values of Youth.* Ann Arbor, Mich.: Institute for Social Research, University of Michigan, 1991.

Barbella, Peter. "Realistic Examples in Elementary Statistics." *Mathematics Teacher* 80 (December 1987).

Barringer, Felicity. "Commerce Dept. Declines to Revise '90 Census Count." *New York Times,* July 16, 1991.

Curriculum and Evaluation Standards for School Mathematics. Reston, Va.: National Council of Teachers of Mathematics, 1989.

Huckabee, David C. "Apportioning Seats in the House of Representatives: The Method of Equal Proportions." Congressional Research Service Report for Congress, 88-143 BOV, Library of Congress, February 1988.

Kidron, Michael, and Ronald Segal. *The New State of the World Atlas.* 4th edition. New York: Simon & Schuster, 1991.

Lee, Felicia R. "No Census Adjustment Hits Big Cities Hard." *New York Times,* July 16, 1991.

Meadows, Donella H. *The Global Citizen.* Washington, D.C.: Island Press, 1991.

Meadows, Donella H., Dennis L. Meadows, Jorgen Randers, and William W. Behrens III. *Limits to Growth.* New York: Universe Books, 1974.

Moore, David S. *Statistics: Concepts and Controversies.* New York: W.H. Freeman and Company, 1991.

Ross, Joan and Marc Ross. "Fermi Problems or How to Make the Most of What You Already Know." *In Estimation and Mental Computation, 1986 Yearbook.* Ed. Harold L. Schoen and Marilyn J. Zweng, Reston, Va.: National Council of Teachers of Mathematics, 1986.

The World Almanac and Book of Facts. New York: Pharros Books, 1990.

World Environment Data Sheet. Washington, D.C.: Population Reference Bureau, 1991.

Government Documents

Congressional District Atlas for the 100th Congress. U.S. Department of Commerce, 1985.

General Social and Economic Characteristics: United States Summary, PC80-1-C1, 1980 Census of Population. Bureau of the Census, U.S. Department of Commerce, 1983.

General Population Characteristics, United States Summary, PC80-1-B1, 1980 Census of Population. Bureau of the Census, U.S. Department of Commerce, 1983.

Journey to Work: Characteristics of Workers in Metropolitan Areas, PC80-2-6D, Section 2, 1980 Census of Population. Bureau of the Census, U.S. Department of Commerce, 1984.

"Why Census Information is Vital to Communities." Bureau of the Census, D-3224.7. U.S. Department of Commerce, Spring 1989.

1989–1990 Official Congressional Directory (S. Prt. 101-29). U.S. Government Printing Office, p. 541.

1990 Census Profile, No. 1 (March 1991). Economics and Statistics Administration, U.S. Department of Commerce.

THE POWER OF NUMBERS

Imagine a math class where the students are fluently speaking the language of mathematics and not waiting for the teacher to tell them everything, where there are lively discussions about what techniques to use, what processes to try, and what the results mean.

In such a class mathematics is alive.

enables students to experience the power and joy of mathematics.

, develops critical-thinking skills in real-world contexts including: designing a public rail transportation system, studying census data, and creating individual polls.

s teaches students to communicate mathematically, apply problem-solving skills and articulate conclusions.

s sets up situations where the students are the questioners, solvers, and sources of their own learning, and where the teacher is observer, guide, and learner.

Clearly the best application of the NCTM *Standards* on the market. The Power Of Numbers is just what middle school math and social studies teachers have been looking for. It is designed to support the unique structure and cooperative learning styles of the middle school classroom, while offering thought-provoking, engaging experiences for students.

James Garvin, Ph.D.
Executive Director,
New England League of Middle Schools

Being able to have discussions in my math class was terrific! I have never had the joy of experiencing that before. Without The r Of Numbers we would have floundered at every step.

Christina Swain
Teacher, Newburyport

I very much like the way the curriculum puts math in its context. This is especially important for female students. s encourages students to think critically about what they read in the news from researchers and pollsters. This is what math is all about. Bravo!

Sundra Flansburg
Associate Director,
Women's Educational Equity Act Publishing Center,
Education Development Center

BEST COPY AVAILABLE